

# PHASES of QCD and HADRONS in MATTER

Wolfram Weise  
Technische Universität München



- PART I:
  - QCD Thermodynamics
  - Symmetry Breaking Pattern
  - Polyakov Loop and Quasiparticles (PNJL Model)
- PART II:
  - Goldstone Bosons and Vector Mesons in Matter
  - (Selected Topics)



# PART I

## understanding and modeling the **PHASES OF QCD**

with

Thomas Hell

Simon Rössner

Claudia Ratti

C. Ratti, M.Thaler, W.Weise: Phys. Rev. **D 73** (2006) 014019

C. Ratti, S. Rößner, M.Thaler, W.Weise: Eur. Phys. J. **C 49** (2007) 213

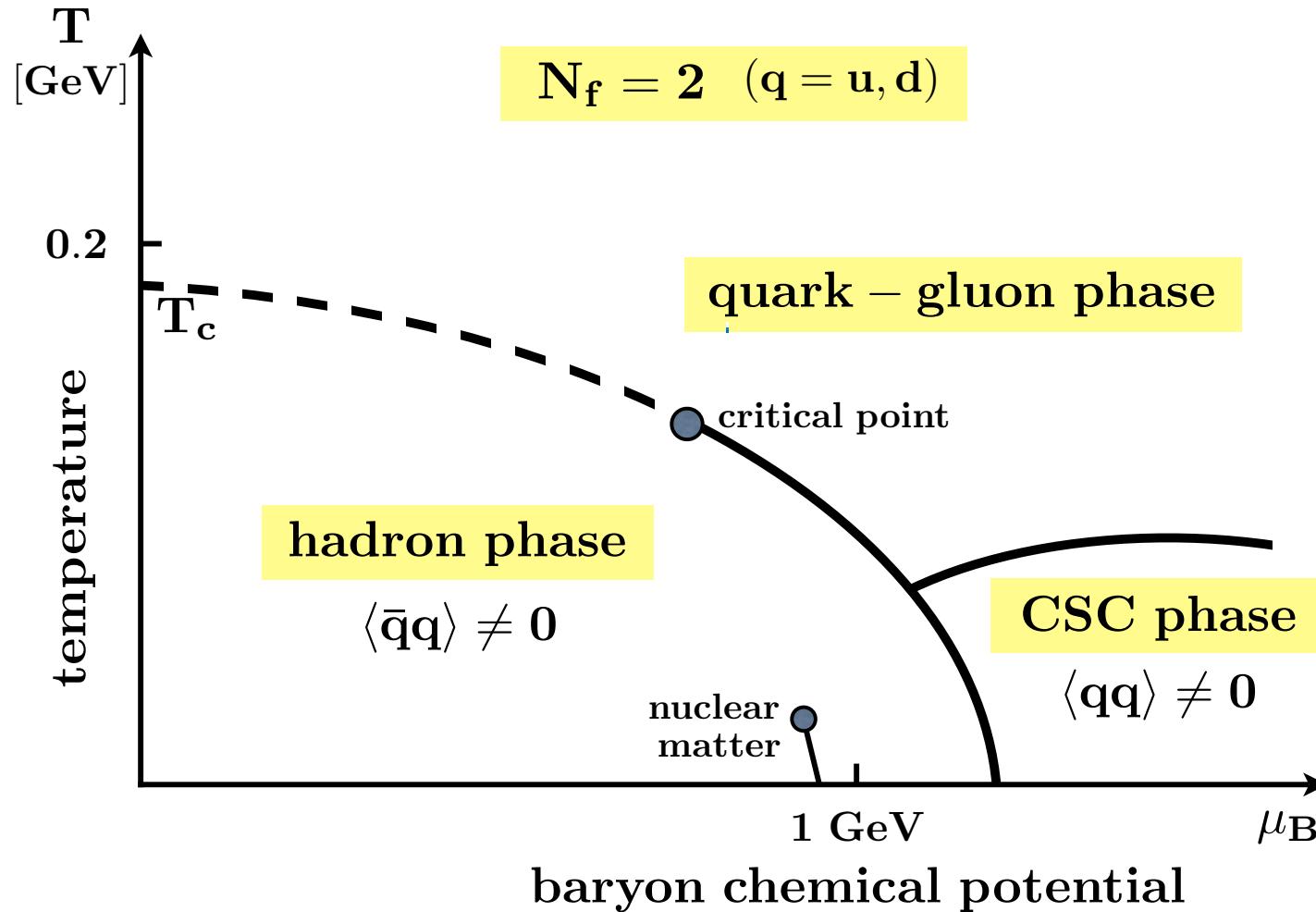
S. Rößner, C. Ratti, W.Weise: Phys. Rev. **D 75** (2007) 034007

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introductory guide through the

# QCD PHASE DIAGRAM

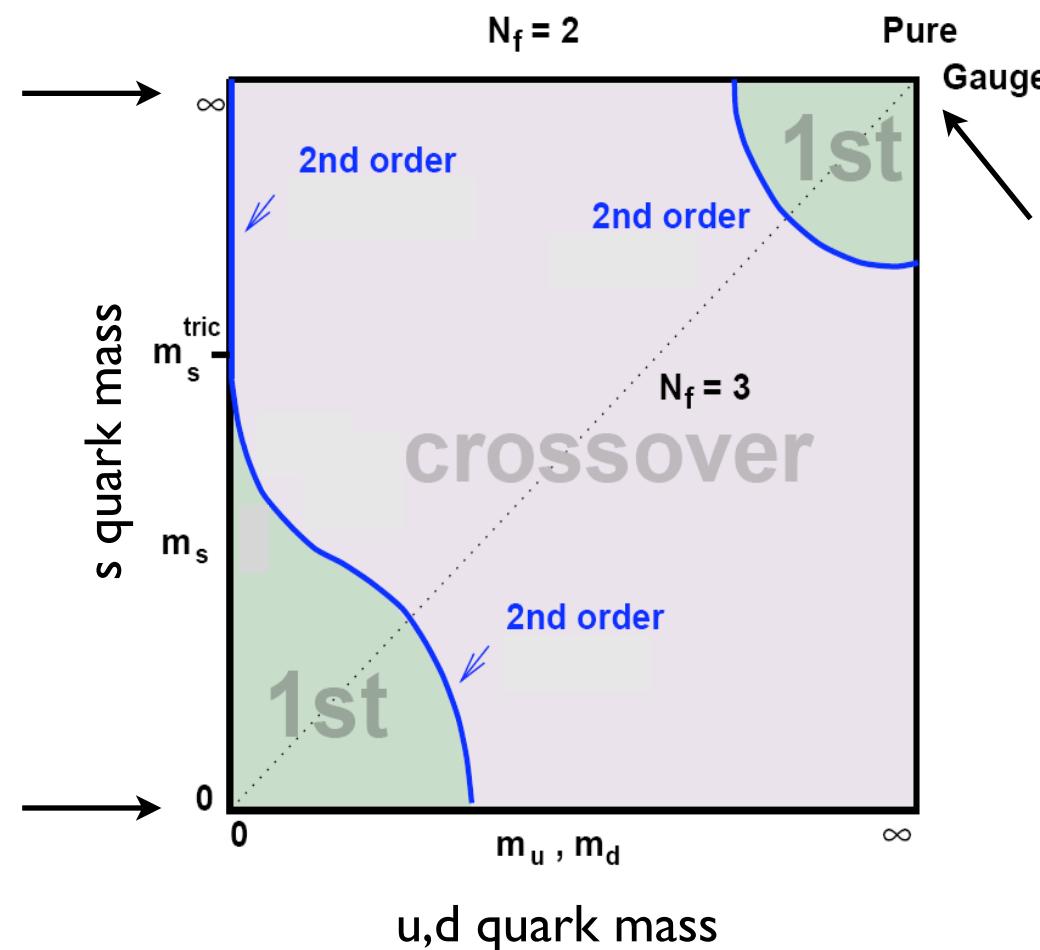


# I. SYMMETRIES and SYMMETRY BREAKING PATTERNS

$SU(2)_R \times SU(2)_L$

CHIRAL  
SYMMETRY

$SU(3)_R \times SU(3)_L$



E. Laermann, O. Philipsen: Ann. Rev. Nucl. Part. Sci. 53 (2003) 163



# I.I $\mathbb{Z}(3)$ SYMMETRY

- Consider PURE GAUGE (“pure glue”) QCD
- Thermodynamics: periodic boundary condition on gauge fields

$$A_\mu(\vec{x}, \tau + \beta) = A_\mu(\vec{x}, \tau) \quad (\beta = 1/T)$$

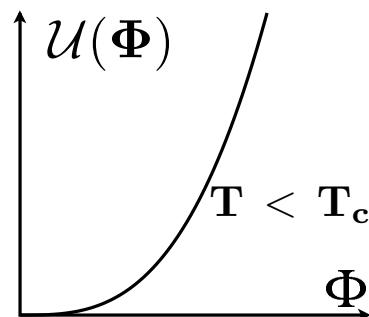
- Gauge transformations:  ${}^g A_\mu = g(A_\mu + \partial_\mu)g^\dagger$

$$g(\vec{x}, \tau + \beta) = \mathbf{z} g(\vec{x}, \tau) \quad \mathbf{z} = \exp\left[i \frac{2\pi n}{N_c}\right] \quad (n = 1, 2, 3, \dots)$$

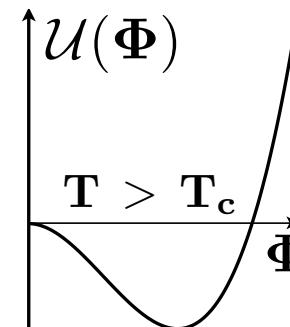
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order parameter

$$\Phi \rightarrow \mathbf{z} \Phi = e^{2\pi i n/3} \Phi$$



**confinement:**  
 $Z(3)$  symmetry  
intact  
 $\langle \Phi \rangle = 0$



**deconfinement:**  
 $Z(3)$  symmetry  
spontaneously  
broken

$$\langle \Phi \rangle \neq 0$$

- QCD with (almost) **MASSLESS u- and d-QUARKS** ( $N_f = 2$ )



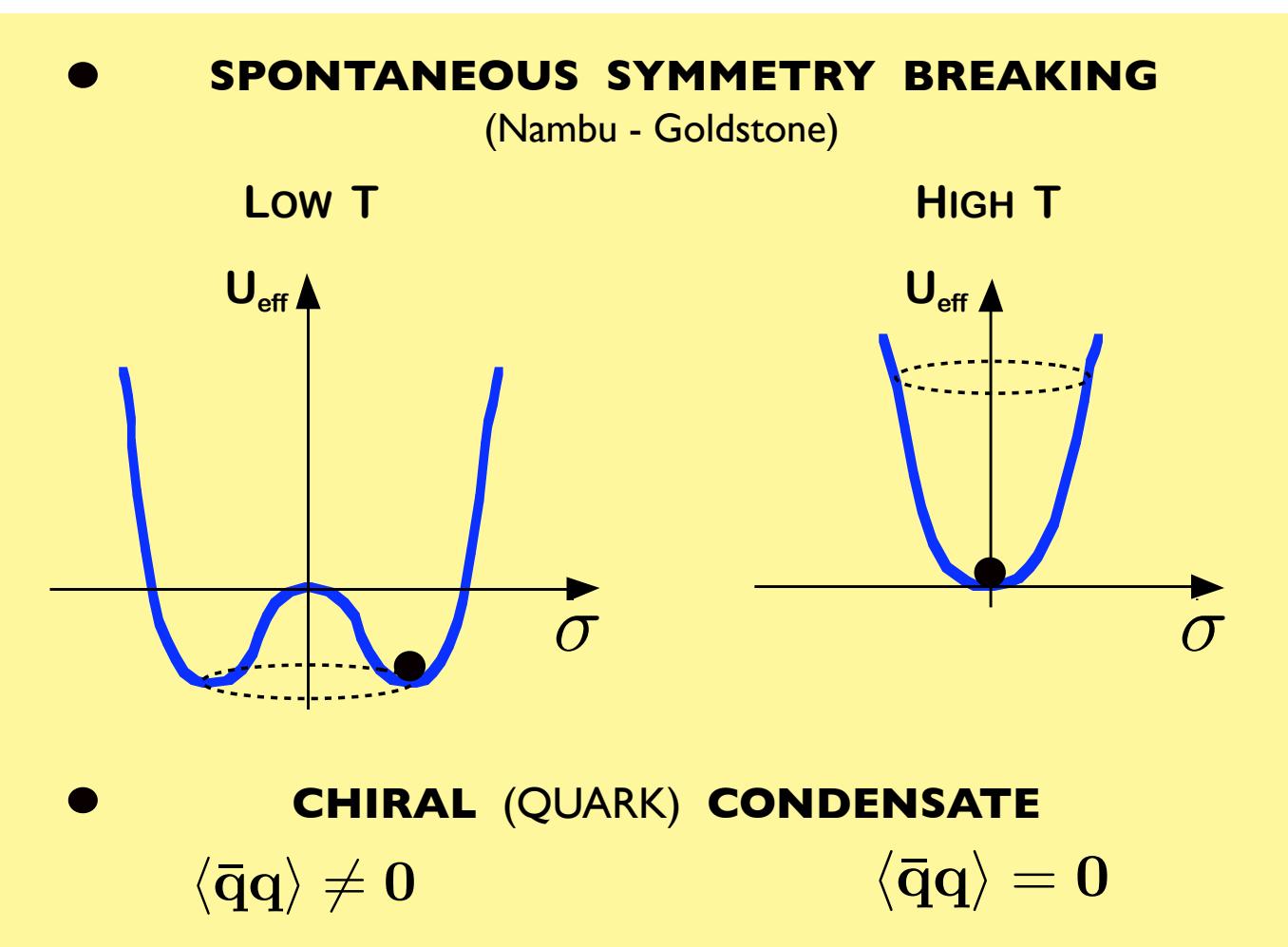
$$\text{SU}(2)_L \times \text{SU}(2)_R$$

$$\psi = (\mathbf{u}, \mathbf{d})^T$$

pseudoscalar - isovector

$$\pi^a \leftrightarrow \bar{\psi} \gamma_5 \mathbf{t}^a \psi$$

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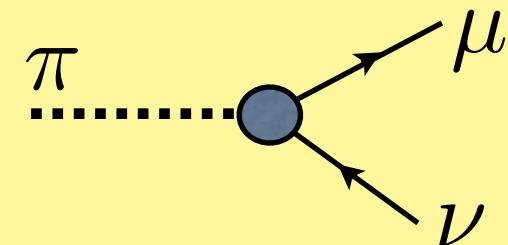
# Spontaneously Broken CHIRAL SYMMETRY

- **NAMBU - GOLDSTONE BOSON: PION**
- **ORDER PARAMETER: PION DECAY CONSTANT**

$$\langle 0 | A_\mu^a(0) | \pi^b(p) \rangle = i\delta^{ab} p_\mu f_\pi$$

Axial current

$$f_\pi = 92.4 \text{ MeV}$$



- **SYMMETRY BREAKING SCALE  $\longleftrightarrow$  MASS GAP**

$$\Lambda_\chi = 4\pi f_\pi \sim 1 \text{ GeV}$$

- **PCAC:**  $m_\pi^2 f_\pi^2 = -m_q \langle \bar{\psi}\psi \rangle + \mathcal{O}(m_q^2)$

Gell-Mann - Oakes - Renner Relation

# SCALES and SYMMETRY BREAKING PATTERN

**HEAVY** versus **LIGHT**

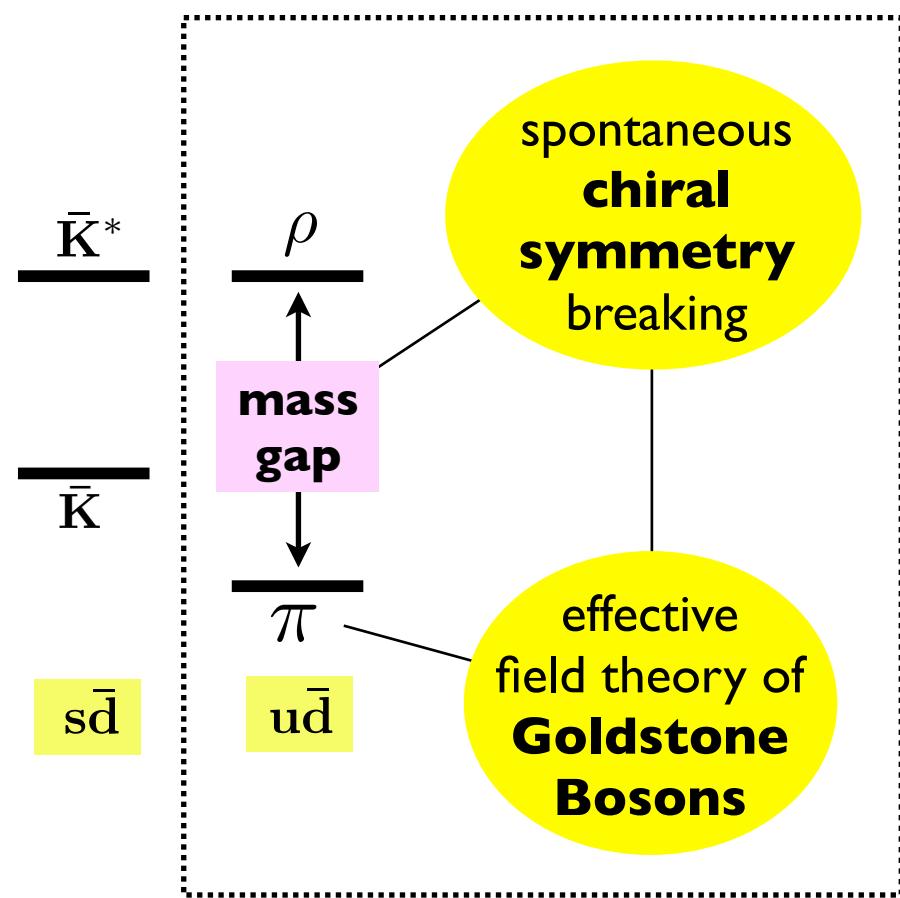
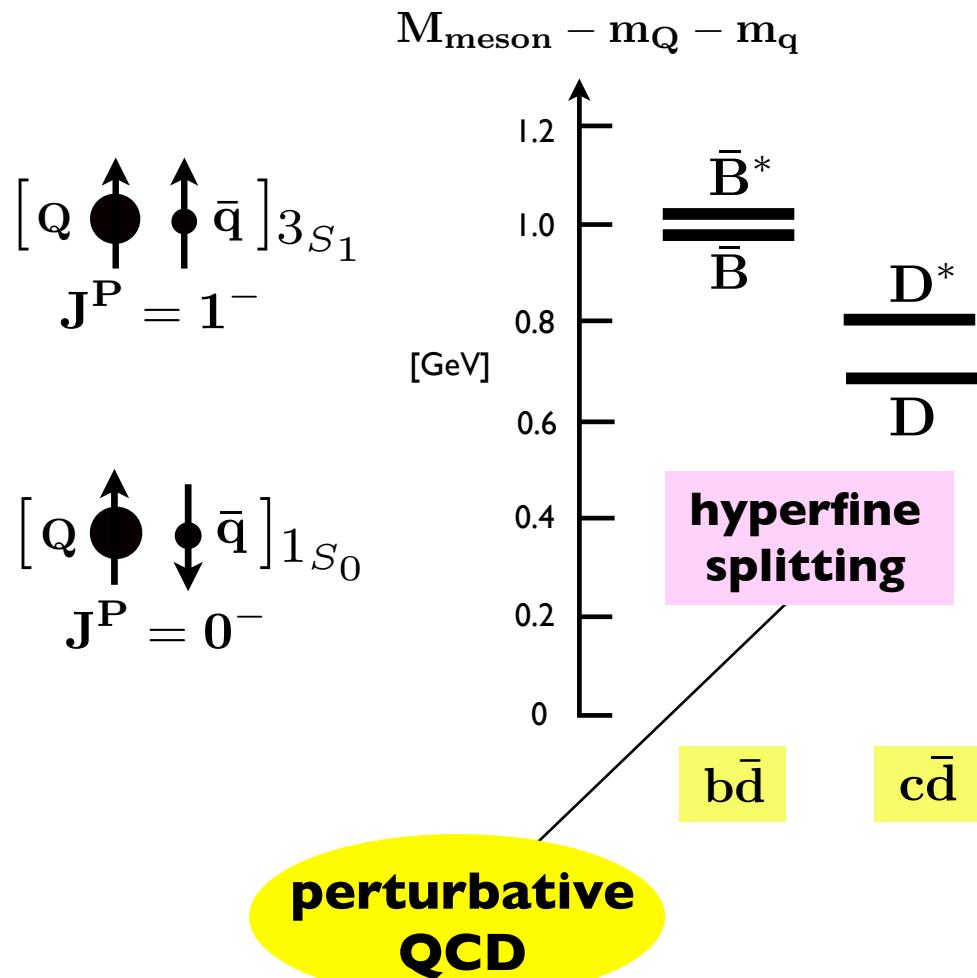
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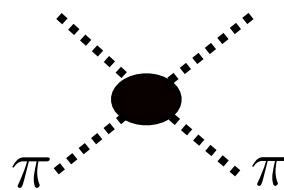
**MASS SPLITTINGS: SINGLET** and **TRIPLET** quark-antiquark states



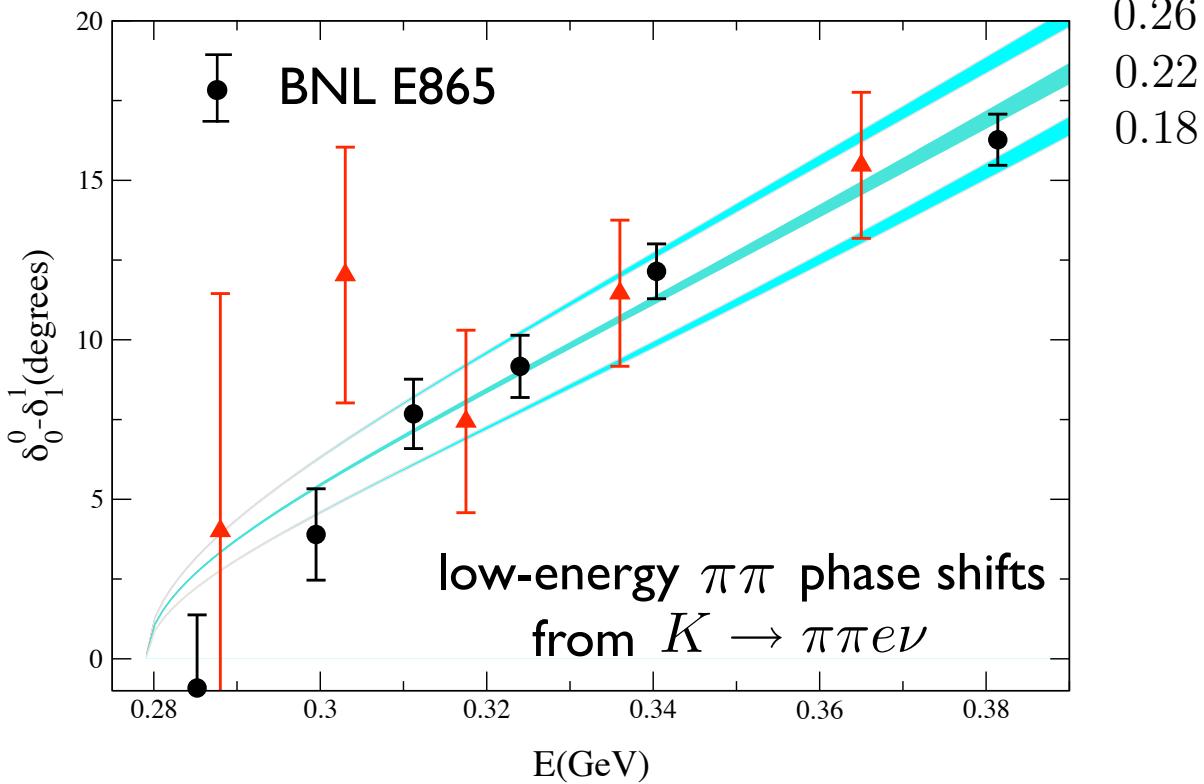
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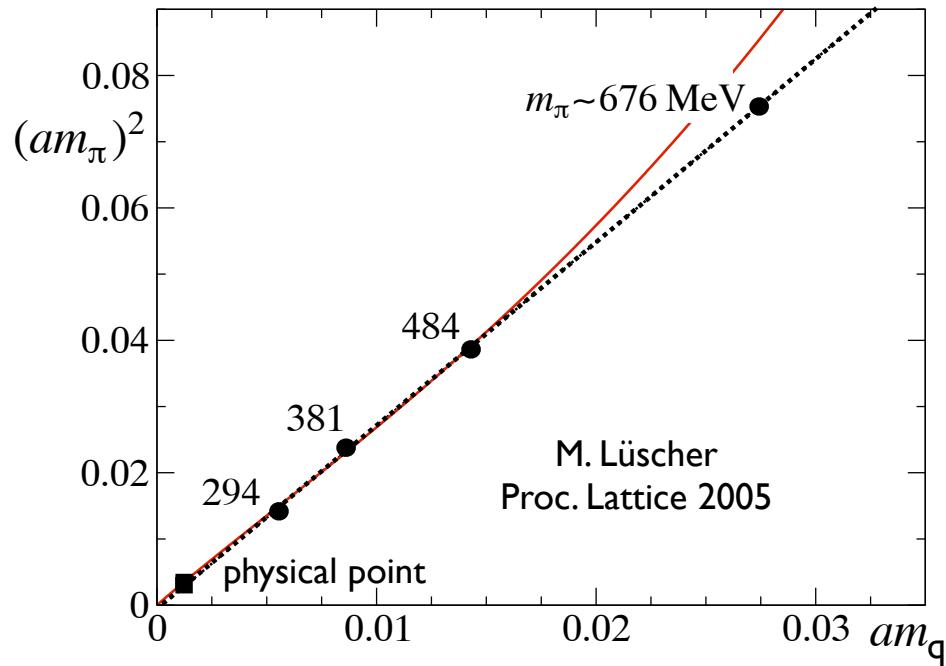
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$$m_\pi^2 = -\frac{m_u + m_d}{f_\pi^2} \langle \bar{q}q \rangle + \mathcal{O}(m_q^2 \log m_q)$$

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$$|\langle \bar{q}q \rangle| \simeq (0.23 \text{ GeV})^3 \simeq 1.5 \text{ fm}^{-3}$$

## 2. Introducing the **PNJL MODEL**

**POLYAKOV LOOP** dynamics

→ **Confinement**

**Synthesis of**

and

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→ **Chiral Symmetry**

... very much like Ginsburg - Landau approach:

identify order parameters as collective degrees of freedom

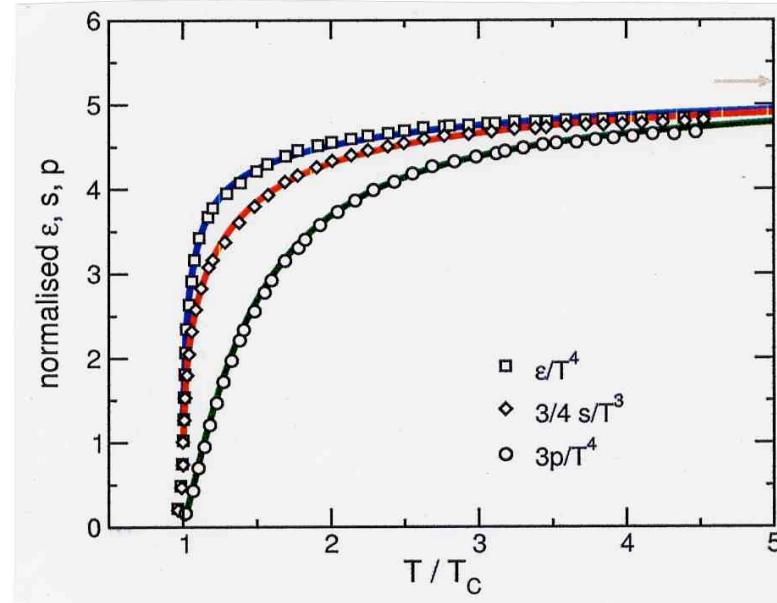
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(e.g.: F. Karsch, J. Phys. G31 (2005) 633)

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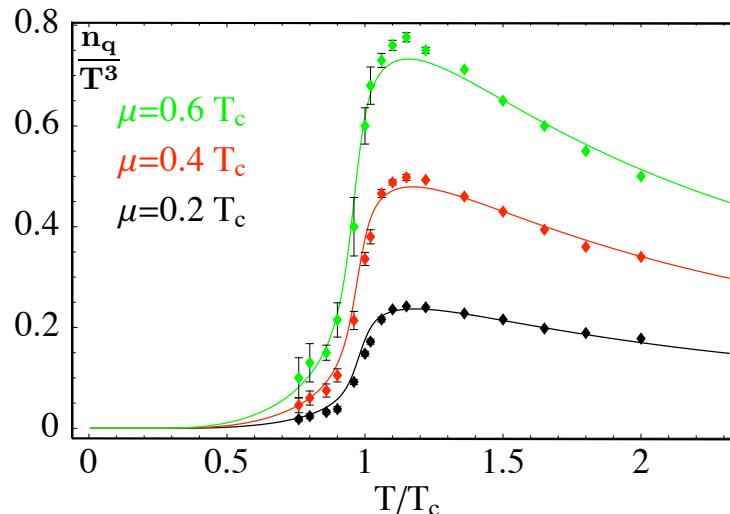
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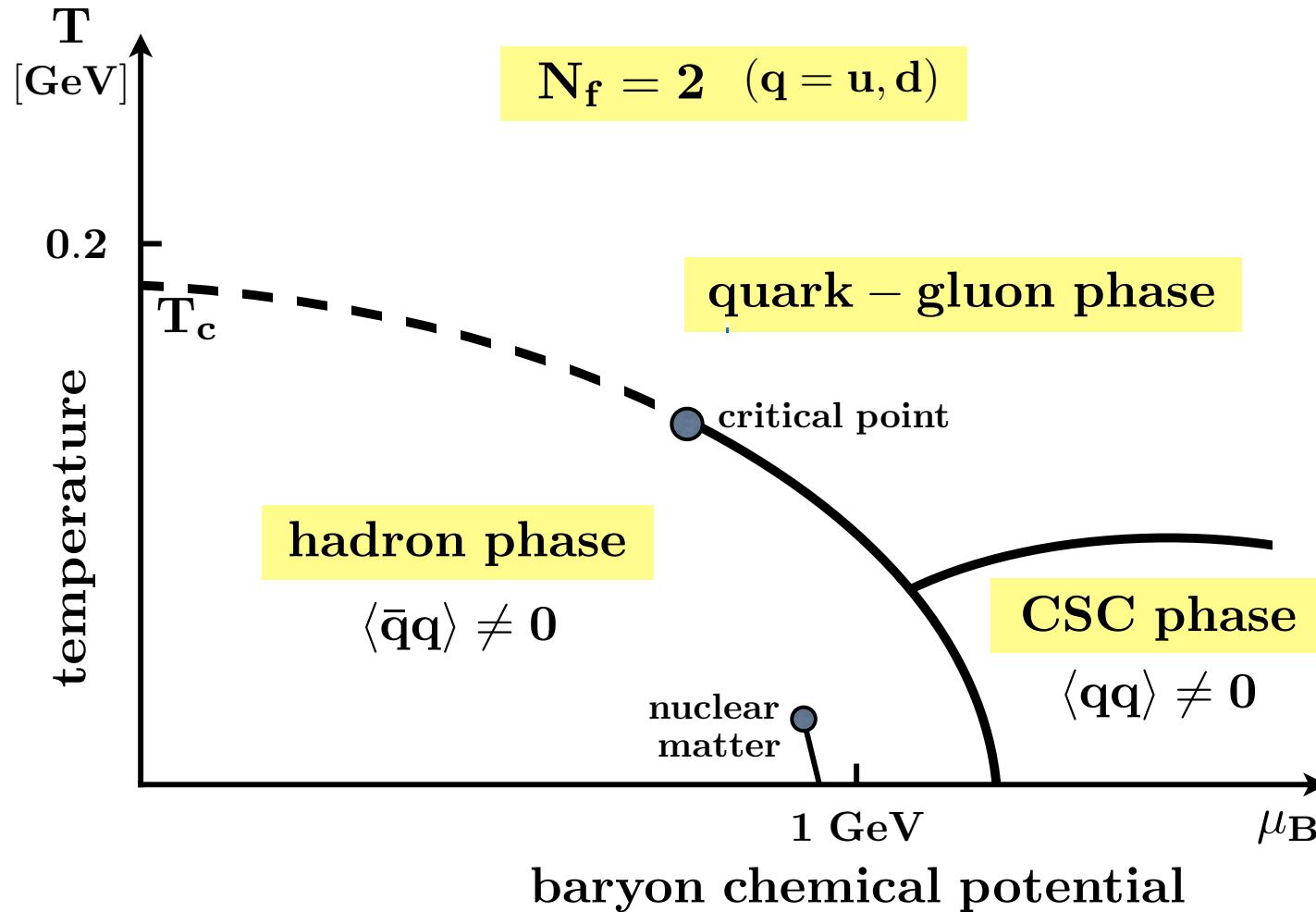
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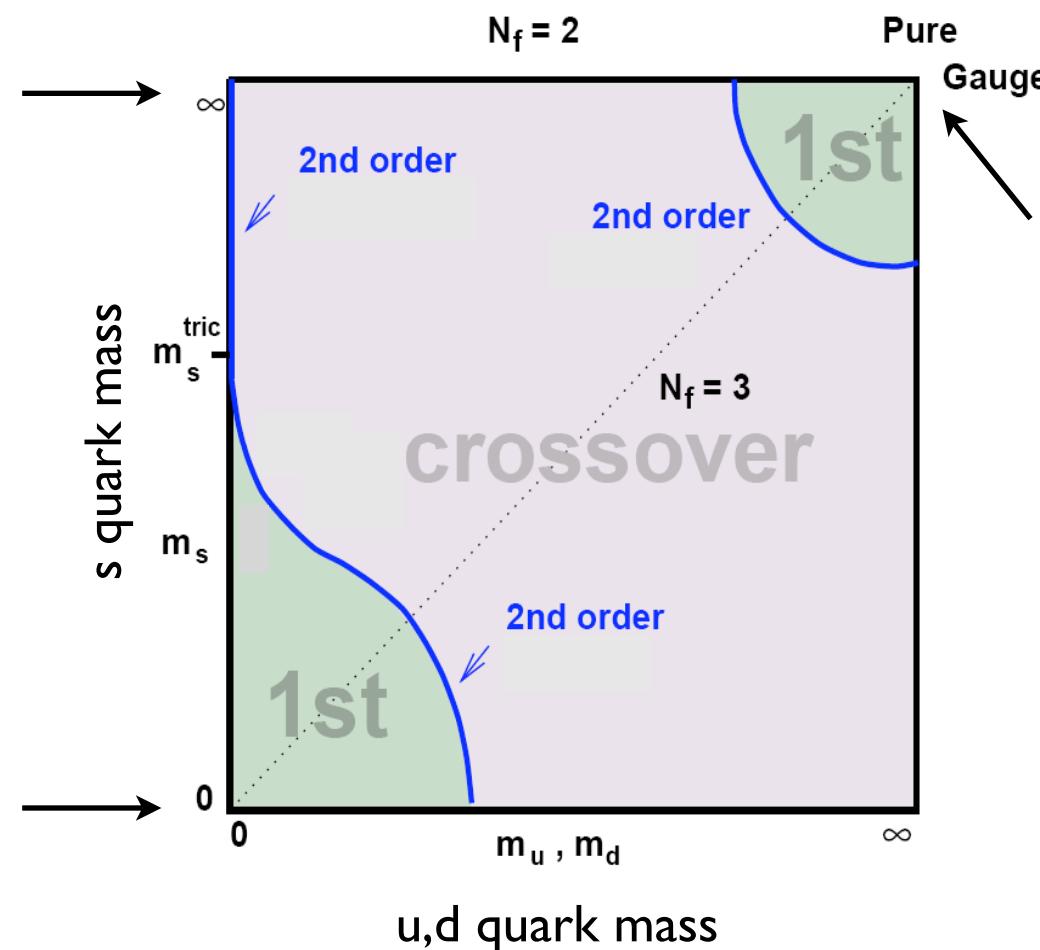


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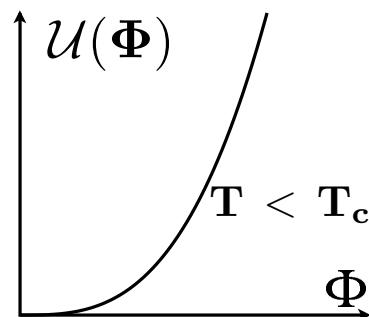
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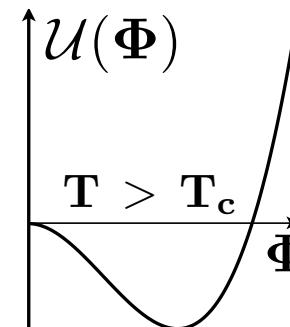
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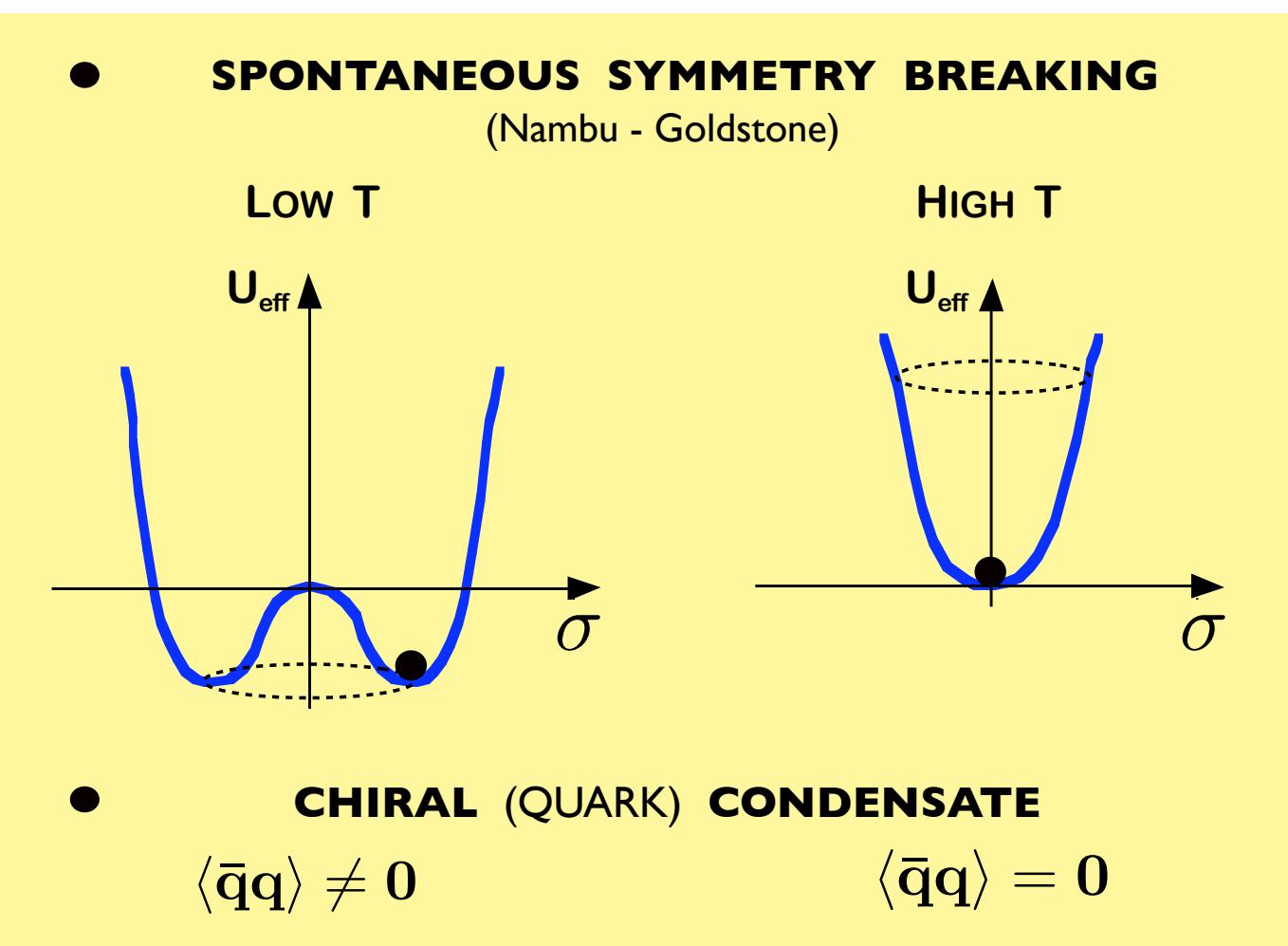
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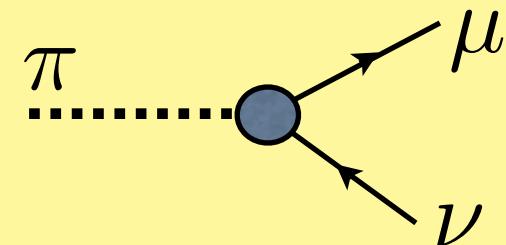
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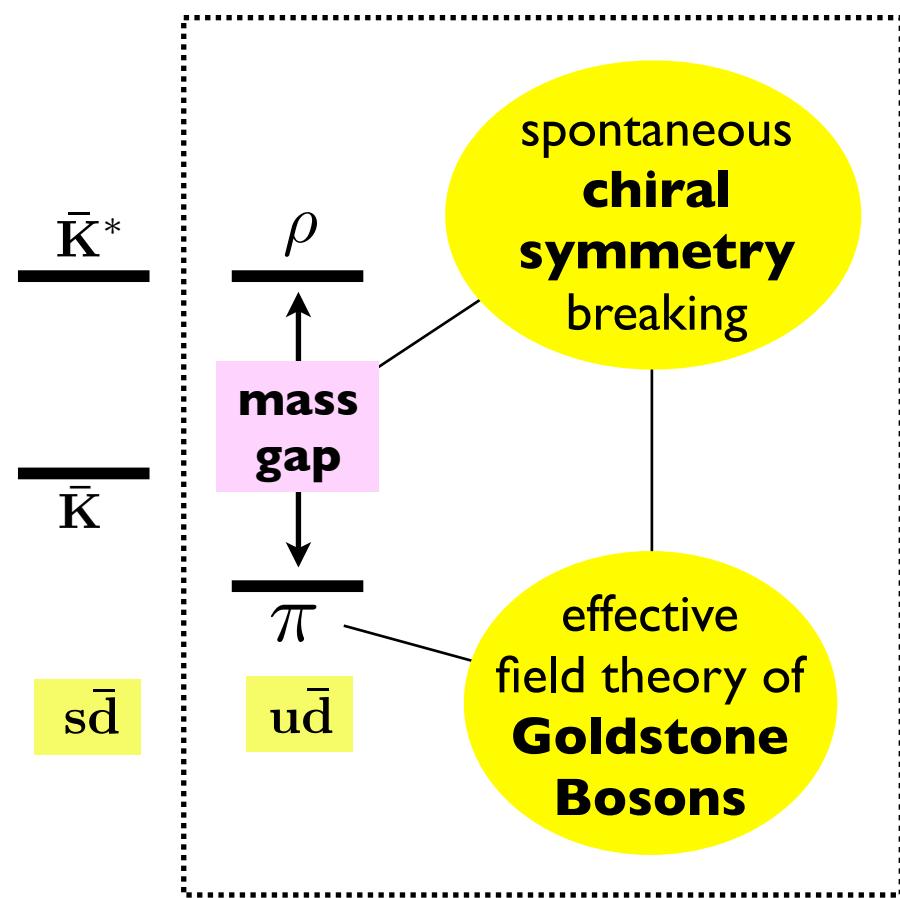
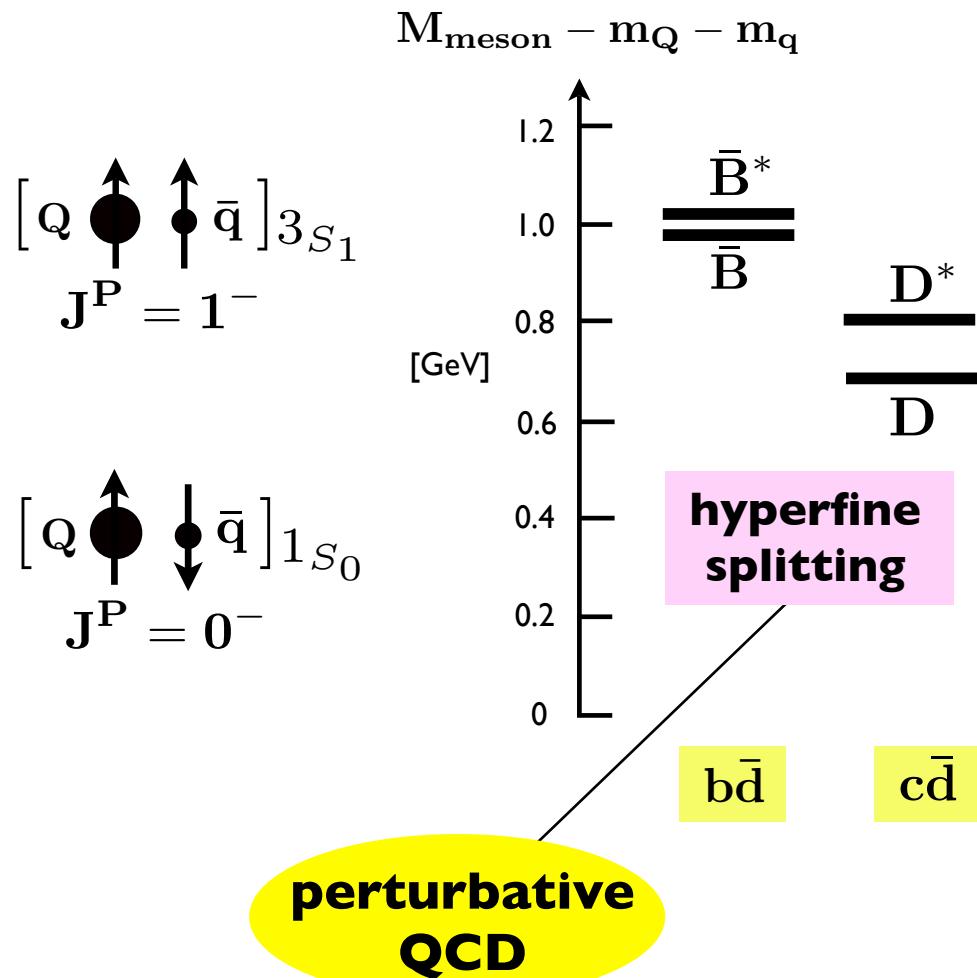
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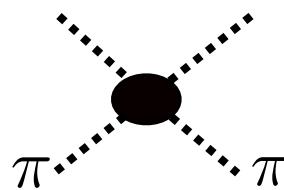
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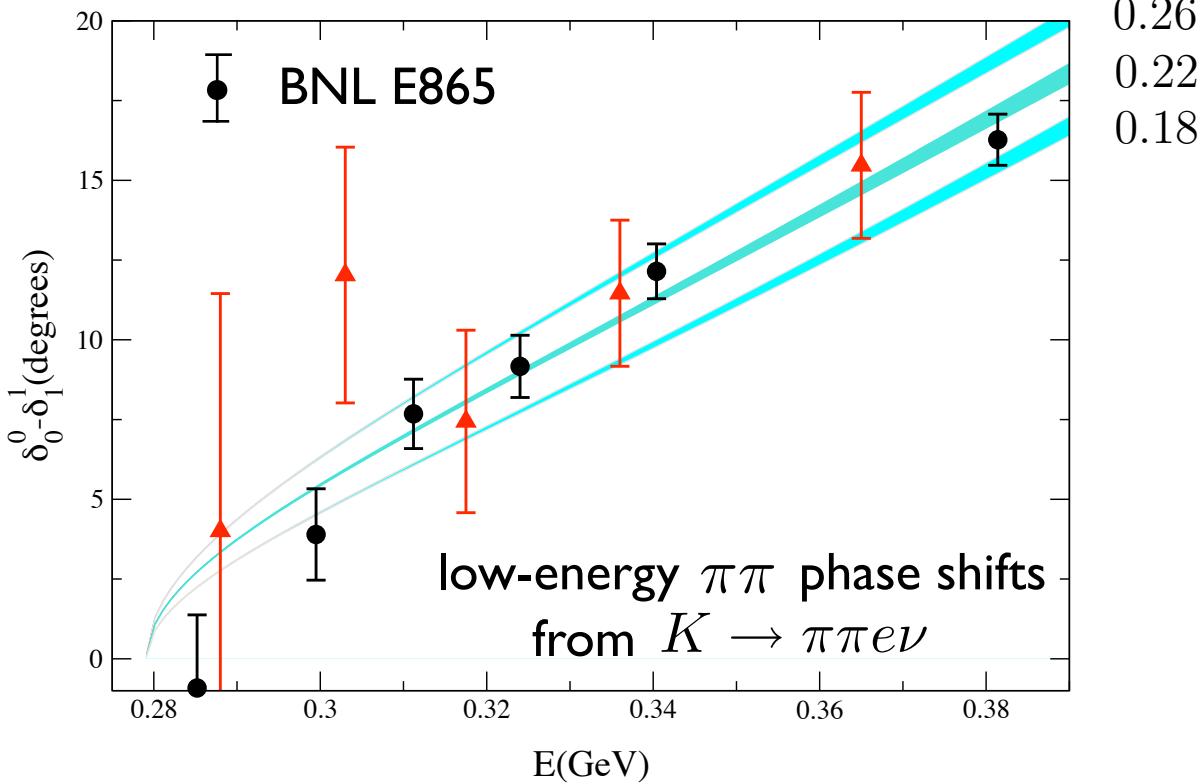
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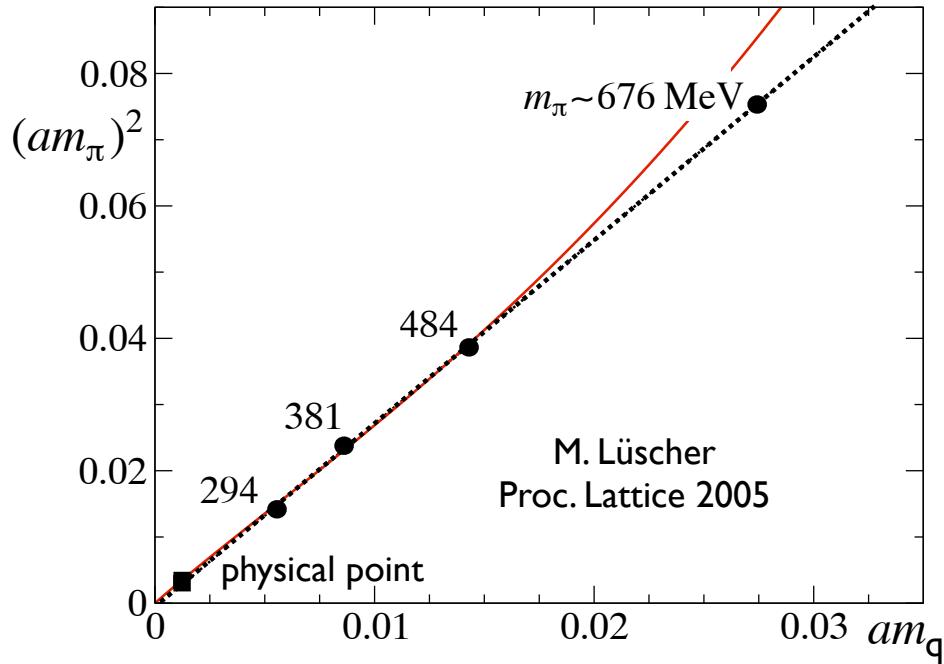
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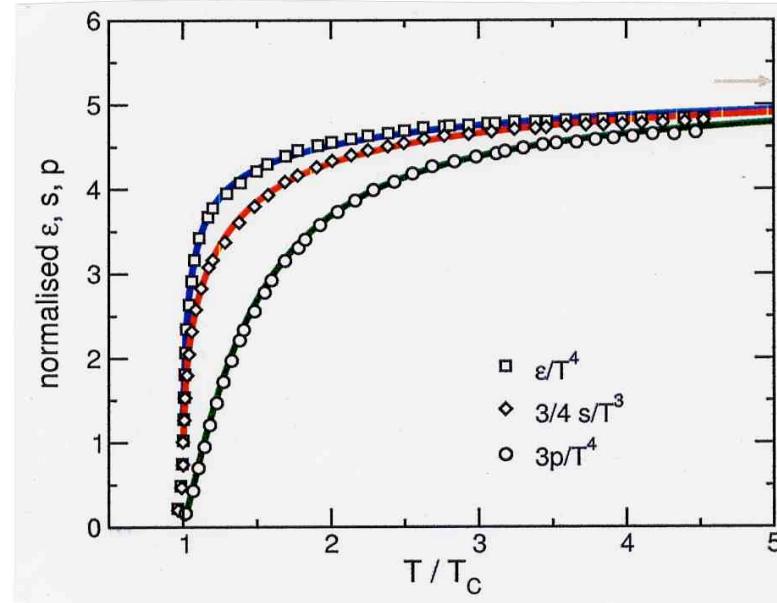


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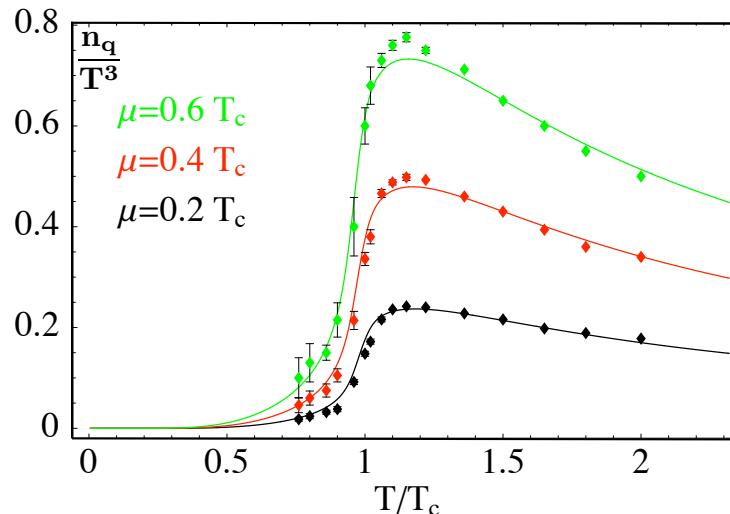
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## 2.2

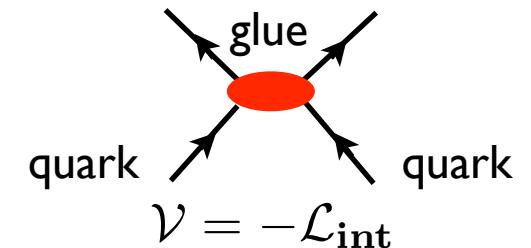
# Sketch of the PNJL MODEL

- **Action :**

$$\mathcal{S}(\psi, \psi^\dagger, \phi) = \int_0^{\beta=1/T} d\tau \int_V d^3x [\psi^\dagger \partial_\tau \psi + \mathcal{H}(\psi, \psi^\dagger, \phi)] - \frac{T}{V} \mathcal{U}(\phi, T)$$

- **Fermionic Hamiltonian density (NJL) :** **Four-Fermion interaction**

$$\mathcal{H} = -i\psi^\dagger (\vec{\alpha} \cdot \vec{\nabla} + \gamma_4 \mathbf{m}_0 - \phi) \psi + \mathcal{V}(\psi, \psi^\dagger)$$

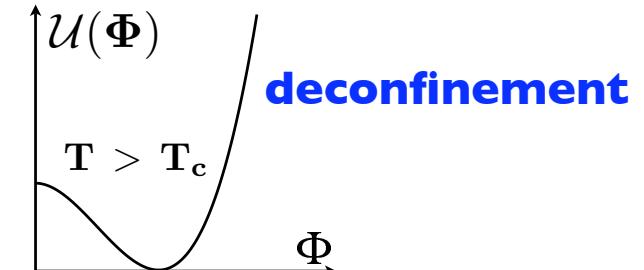
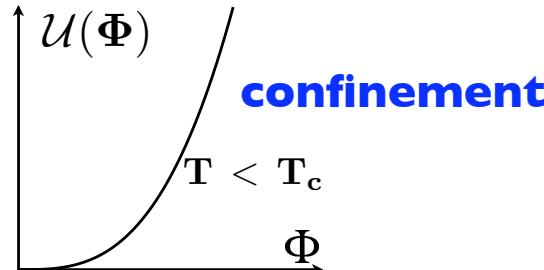


- **Temporal background gauge field**  $\phi = \phi_3 \lambda_3 + \phi_8 \lambda_8 \in \text{SU}(3)$

$$\Phi = \frac{1}{N_c} \text{Tr} \left[ \exp \left( i \int_0^{1/T} d\tau A_4 \right) \right] \equiv \frac{1}{3} \text{Tr} \exp(i\phi/T)$$

**Polyakov loop**

- **Effective potential :**



## 2.3

# Basics of the **NJL MODEL**

Y. Nambu, G. Jona-Lasinio: Phys. Rev. 122 (1961) 345

... applications to

## **HADRON PHYSICS:**

U.Vogl, W.W.: Prog. Part. Nucl. Phys. 27 (1991) 195

T. Hatsuda, T. Kunihiro: Phys. Reports 247 (1994) 221

+ many others

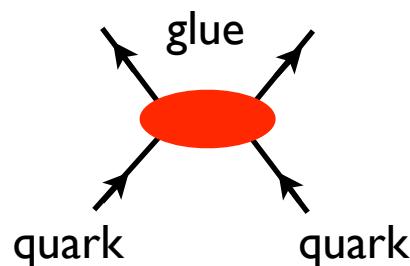
- **QUARK COLOR CURRENT:**

$$\mathbf{J}_\mu^a(x) = \bar{\psi}(x) \gamma^\mu \frac{\lambda^a}{2} \psi(x)$$

- Assume: **short correlation range** for “**color transport**” between quarks

$$l_c < 0.2 \text{ fm}$$

$$G_c \sim g^2 l_c^2$$



$$\mathcal{L}_{int} = -G_c \mathbf{J}_\mu^a(x) \mathbf{J}_a^\mu(x)$$

**(chiral invariant)**

**LOCAL**  $SU(N_c)$   
gauge symmetry of  
**QCD**



**GLOBAL**  $SU(N_c)$   
symmetry of  
**NJL** model

- Fierz transform of Color - Current-Current Interaction ( $N_f = 2$  flavors):

→ **QUARK-ANTIQUARK channels**

$$\mathcal{L}_{q\bar{q}} = \frac{G}{2} [(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] + \dots$$

vector + axial vector  
+ color octet terms

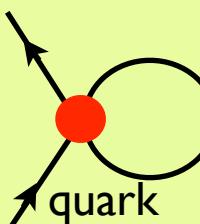
→ **DIQUARK channels**     $\mathcal{L}_{qq} = \frac{H}{2} (\bar{\psi}i\gamma_5\tau_2\lambda^A C\bar{\psi}^T)(\psi^T C i\gamma_5\tau_2\lambda^A \psi) + \dots$

- Self-consistent **MEAN FIELD** approximation:

→ **GAP equation:**

$$M = m_o - G\langle\bar{\psi}\psi\rangle$$

$$G = \frac{4}{3}H \simeq 10 \text{ GeV}^{-1}$$



**CHIRAL CONDENSATE:**

$$\langle\bar{\psi}\psi\rangle = -2iN_f N_c \int \frac{d^4 p}{(2\pi)^4} \frac{M \theta(\Lambda^2 - \vec{p}^2)}{p^2 - M^2 + i\epsilon}$$

$$m_0 \simeq 5 \text{ MeV}$$

$$\Lambda \simeq 0.65 \text{ GeV}$$

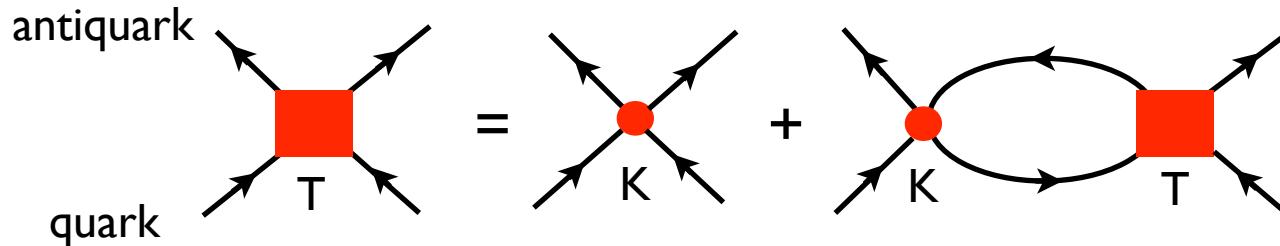
- **SPONTANEOUS CHIRAL SYMMETRY BREAKING**  
 → **GOLDSTONE BOSONS (pions)**

...but:

**no confinement !**

# the MESON sector

- Bethe-Salpeter Equation in (colour singlet) QUARK-ANTIQUARK channels:



- SU(3) NJL model including Axial U(1) breaking 't Hooft interaction (INSTANTONS)

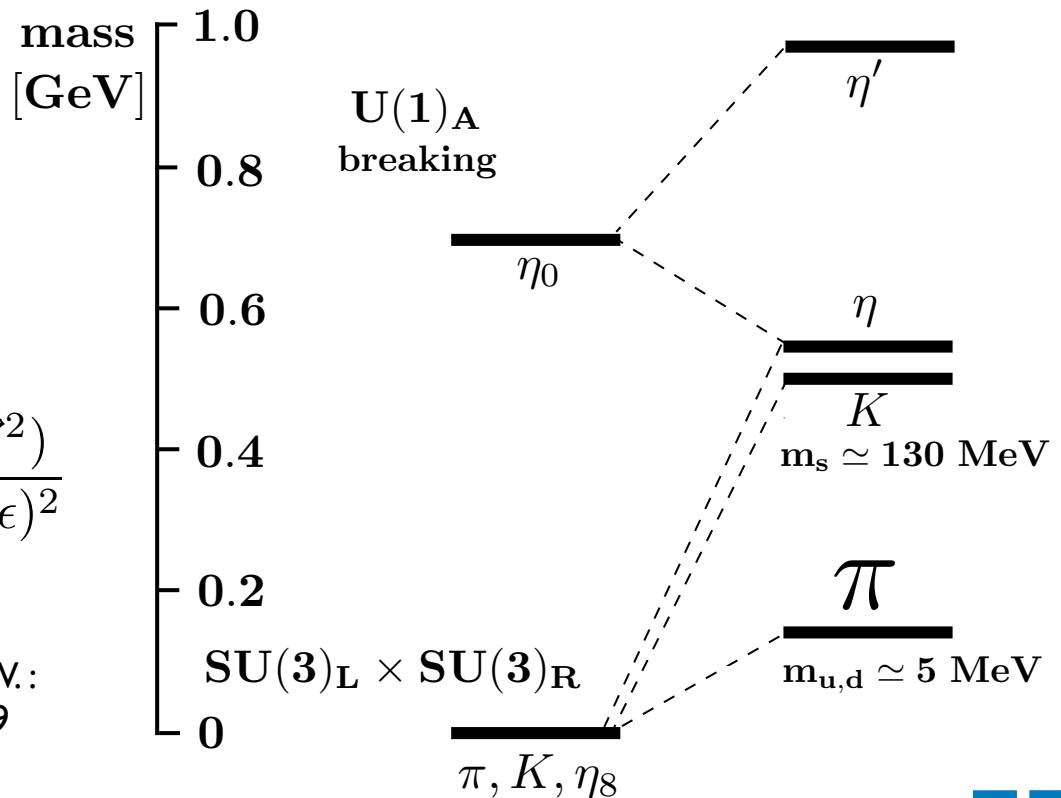
PSEUDOSCALAR MESON SPECTRUM

- Gell-Mann - Oakes -Renner Relation satisfied:

$$m_\pi^2 = -\frac{m_u + m_d}{f_\pi^2} \langle \bar{q}q \rangle + \mathcal{O}(m_q^2)$$

$$f_\pi^2 = -4iN_c \int \frac{d^4 p}{(2\pi)^4} \frac{M^2 \theta(\Lambda^2 - p^2)}{(p^2 - M^2 + i\epsilon)^2}$$

S. Klimt, M. Lutz, U. Vogl, W.W.:  
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## 2.3 Polyakov Loop (Thermal Wilson Line)

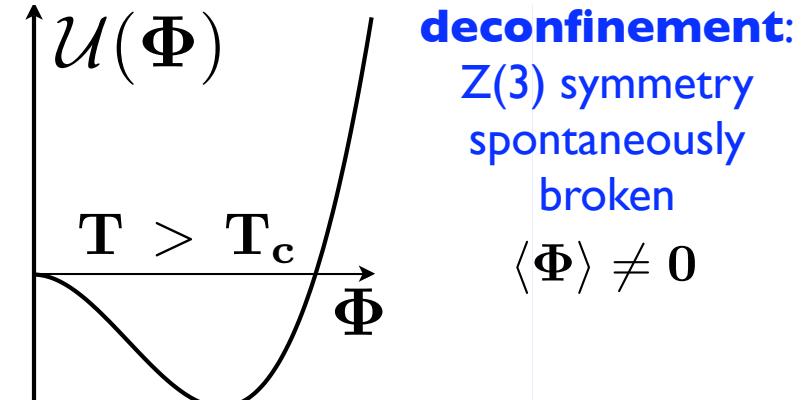
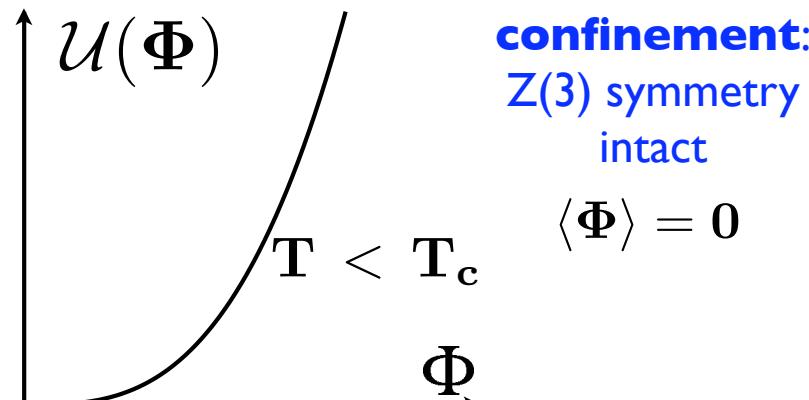
- Order parameter of spontaneously broken  $Z(N_c)$  center symmetry of  $SU(N_c)$  pure gauge theory ( $\rightarrow$  **DECONFINEMENT**)

$$\Phi = \frac{1}{N_c} \text{Tr} \left[ \exp \left( i \int_0^{1/T} d\tau A_4 \right) \right] \equiv \underbrace{\frac{1}{3} \text{Tr} \exp(i\phi/T)}_{\in SU(3)}$$

$$\phi = \phi_3 \lambda_3 + \phi_8 \lambda_8$$

- Effective potential :** (R. Pisarsky (2000); K. Fukushima (2004))

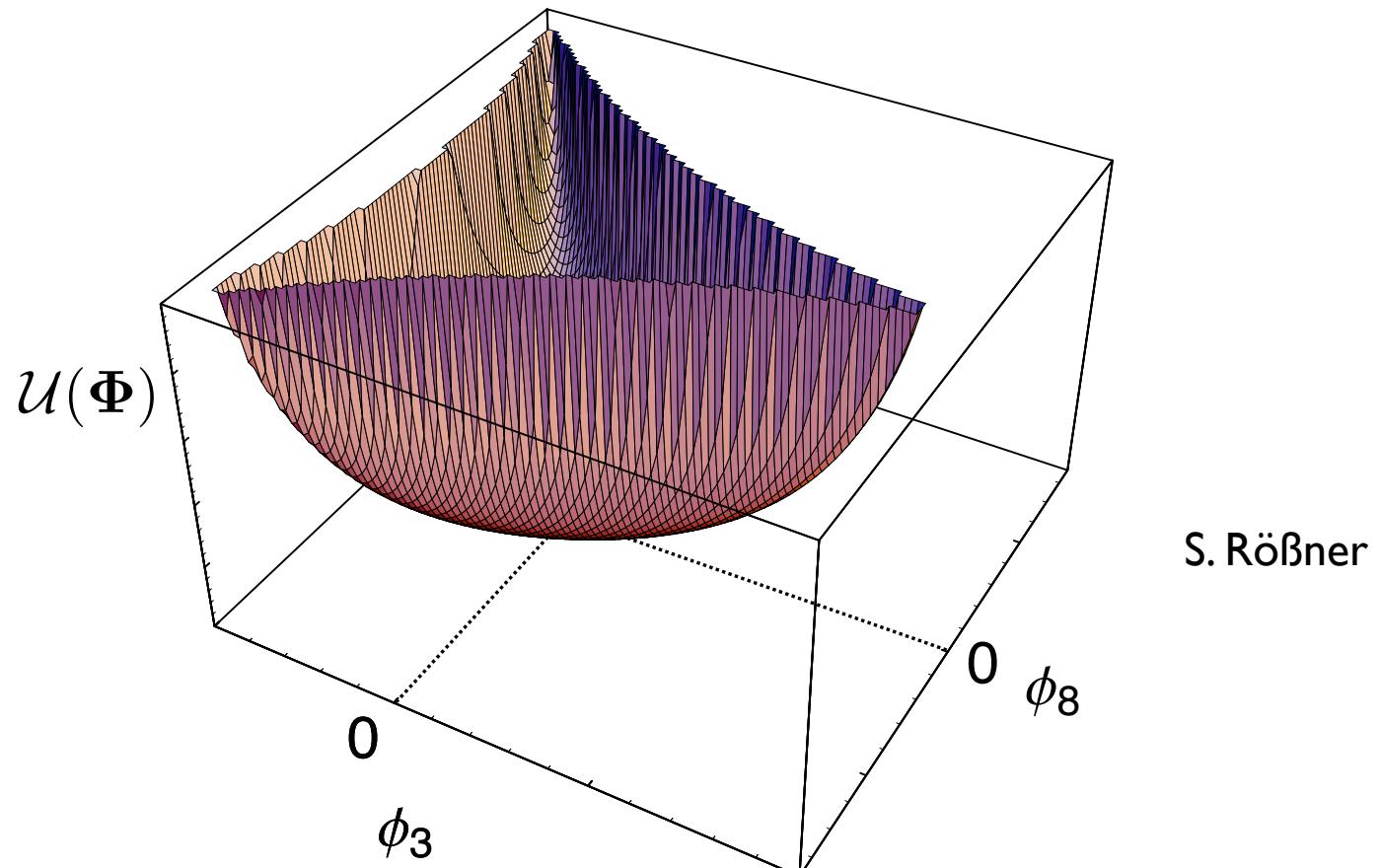
$$U(\Phi, T) = -\frac{1}{2} a(T) \Phi^* \Phi - b(T) \ln[1 - 6 \Phi^* \Phi + 4(\Phi^*{}^3 + \Phi^3) - 3(\Phi^* \Phi)^2]$$



# Polyakov Loop EFFECTIVE POTENTIAL and $\mathbb{Z}(3)$ SYMMETRY

$$\Phi \rightarrow z \Phi = e^{2\pi i n/3} \Phi \quad (n = 1, 2, 3, \dots)$$

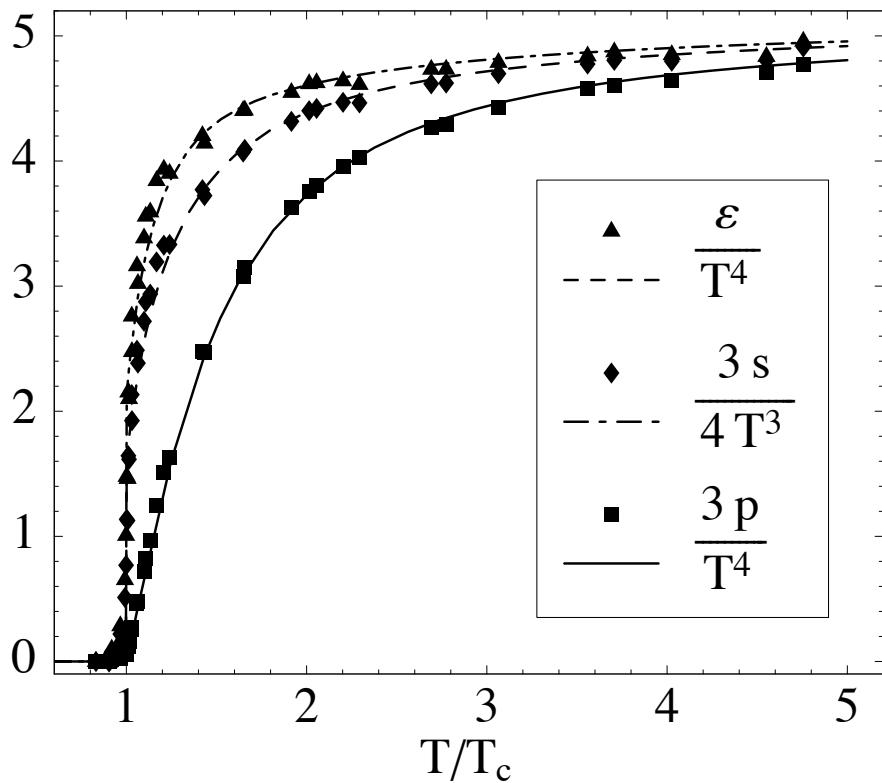
$$\mathcal{U}(\Phi, T) = -\frac{1}{2}a(T) \Phi^* \Phi - b(T) \ln[1 - 6 \Phi^* \Phi + 4(\Phi^*{}^3 + \Phi^3) - 3(\Phi^* \Phi)^2]$$



# Comparison with “PURE GLUE” Lattice Thermodynamics

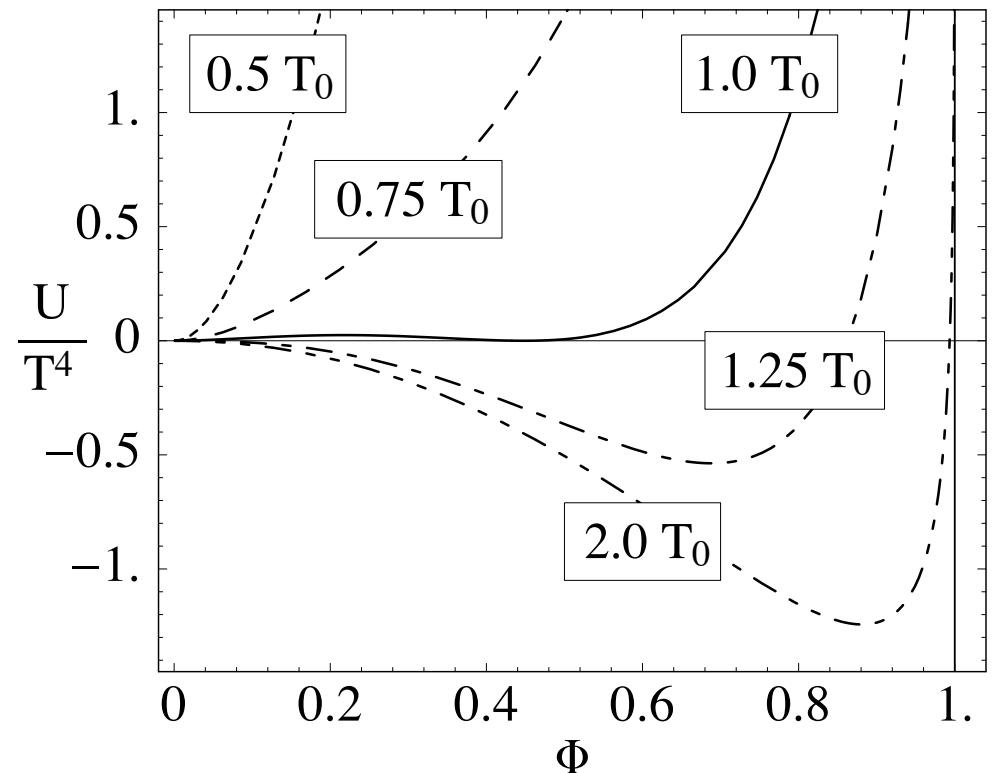
- Minimization of  $\mathcal{U}(\Phi(T), T) = -p(T)$   $\longrightarrow$  fit  $a(T), b(T)$

**energy density, entropy density,  
pressure**



O. Kaczmarek et al.  
Phys. Lett. B 543 (2002) 41

**Effective potential**



S. Rößner, C. Ratti, W.W.  
Phys. Rev. D 75 (2007) 034007

**first order phase transition**

$T_c(\text{pure gauge}) \equiv T_0 \simeq 270 \text{ MeV}$

## 2.4

# Thermodynamics of the PNJL Model

- Action :

$$\mathcal{S} = \int_0^{\beta=1/T} d\tau \int_V d^3x [\psi^\dagger \partial_\tau \psi + \mathcal{H}(\psi, \psi^\dagger, \phi)] - \frac{T}{V} \mathcal{U}(\phi, T)$$

- Bosonisation:

→ Goldstone Bosons of spontaneously broken Chiral Symmetry

$$\pi_a \leftrightarrow \bar{\psi} i\gamma_5 \tau_a \psi$$

→ Scalar Field  $\sigma \leftrightarrow \bar{\psi} \psi$  Chiral Condensate  $\langle \sigma \rangle \leftrightarrow \langle \bar{\psi} \psi \rangle$

→ Diquark Field  $\Delta \leftrightarrow \psi^T \Gamma \psi$  Cooper Pair Condensate  $\langle \Delta \rangle \leftrightarrow \langle \psi^T \Gamma \psi \rangle$

- Thermodynamical Potential:

$$\Omega = -\frac{T}{V} \ln Z \quad Z = \int \mathcal{D}\varphi \exp[-\mathcal{S}(T, V, \mu)] \quad (\partial_\tau \rightarrow \partial_\tau - \mu)$$

# Thermodynamics of the PNJL Model

- Thermodynamical Potential after Matsubara sums:

$$\Omega = \mathcal{U}(\Phi, T) + \frac{\sigma^2}{2G} + \frac{\Delta^* \Delta}{2H} - 2N_f \int \frac{d^3 p}{(2\pi)^3} \sum_j T \ln \left[ 1 + e^{-E_j/T} \right] + \text{const}$$

- Quasiparticle “branches”:

$$E_{1,2} = \varepsilon(\vec{p}) \pm \tilde{\mu}_b ,$$

$$E_{3,4} = \sqrt{(\varepsilon(\vec{p}) + \tilde{\mu}_r)^2 + |\Delta|^2} \pm i \phi_3 ,$$

$$E_{5,6} = \sqrt{(\varepsilon(\vec{p}) - \tilde{\mu}_r)^2 + |\Delta|^2} \pm i \phi_3 ,$$

$$\varepsilon(\vec{p}) = \sqrt{\vec{p}^2 + m^2} \quad m = m_0 - \sigma = m_0 - G \langle \bar{\psi} \psi \rangle$$

$$\tilde{\mu}_b = \mu + 2i \frac{\phi_8}{\sqrt{3}} , \quad \tilde{\mu}_r = \mu - i \frac{\phi_8}{\sqrt{3}} .$$

- Minimize Thermodynamical Potential  $\rightarrow$  Mean Field Equations:

$$\frac{\partial \text{Re } \Omega}{\partial \varphi} = 0$$

$(\varphi = \sigma, \Delta, \phi_3, \phi_8)$

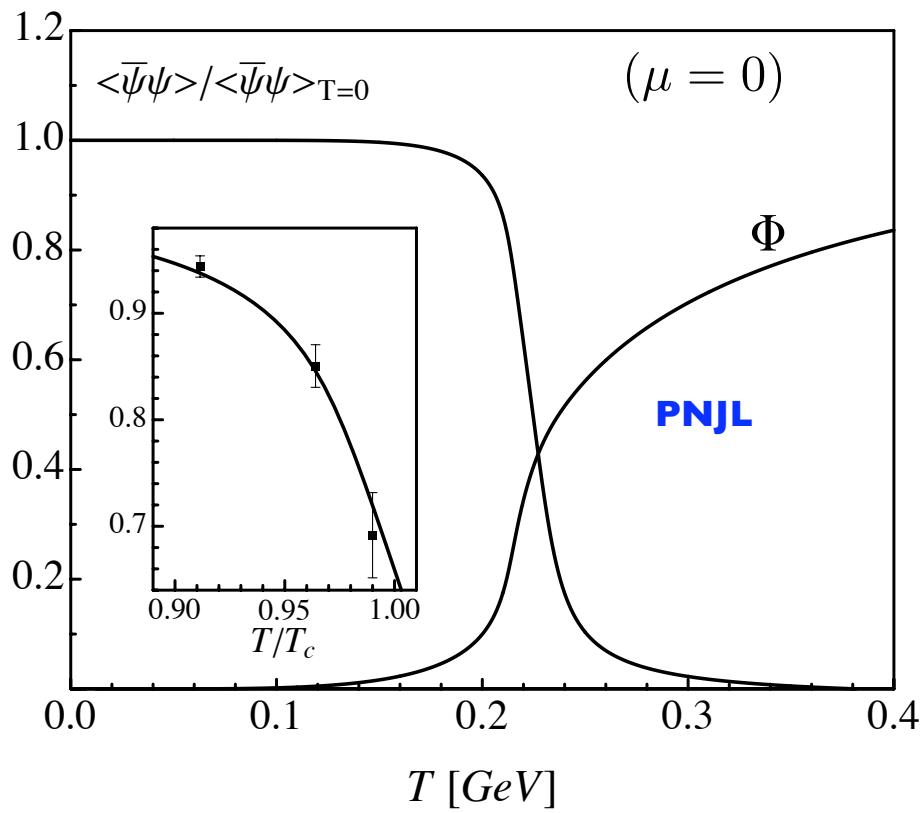
chiral condensate

diquark

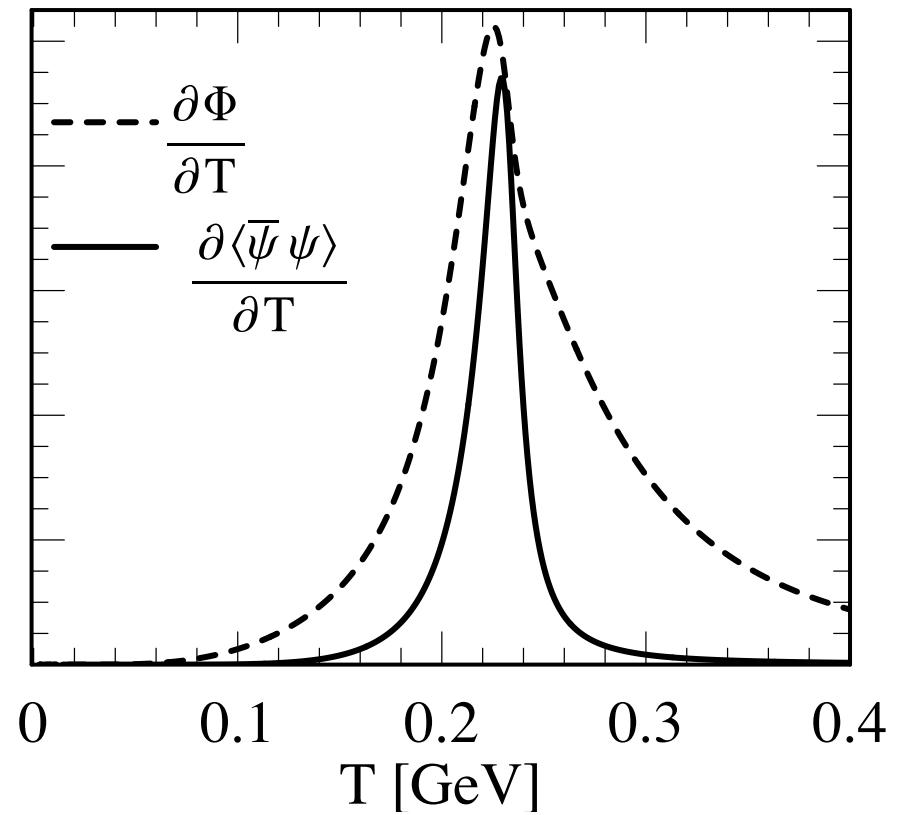
Polyakov loop  
generators

### 3. Results

## PART I: DECONFINEMENT and CHIRAL SYMMETRY RESTORATION



**CHIRAL** and  
**DECONFINEMENT**  
transitions  
almost coincide !



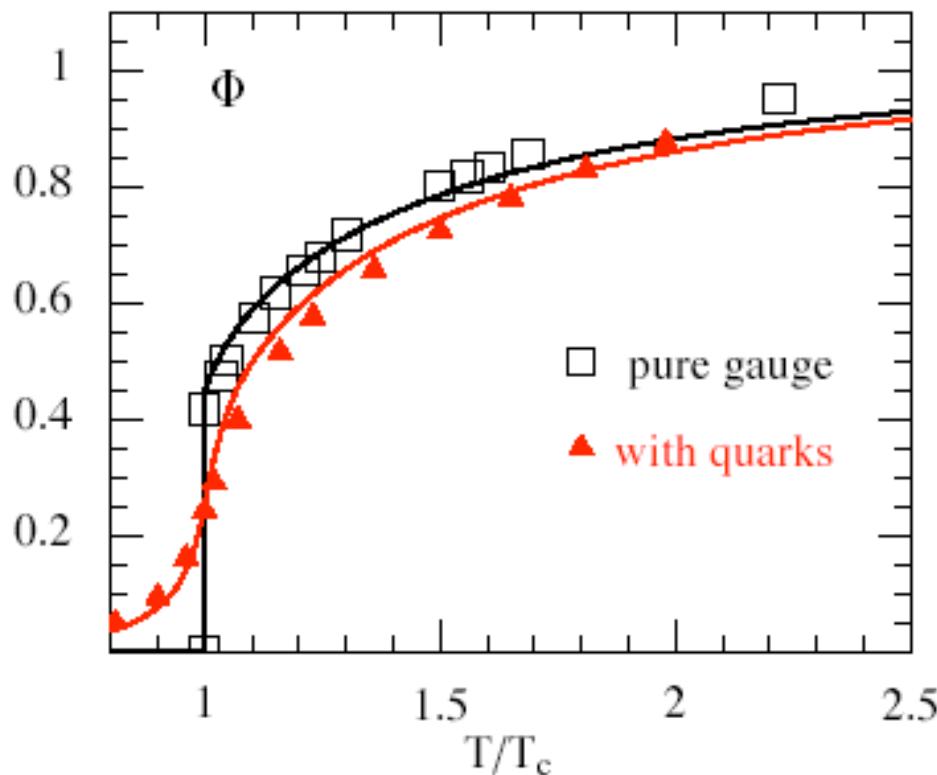
**PNJL:**  
S. Rößner, C. Ratti, W.W.:  
Phys. Rev. **D 75** (2007) 034007

**Lattice results:**  
G. Boyd et al., Phys. Lett. **B 349** (1995) 170

# POLYAKOV LOOP

at zero chemical potential

- PNJL prediction in comparison with 2-flavour Lattice QCD Thermodynamics



S. Rößner, C. Ratti, W.W.  
Phys. Rev. D 75 (2007) 034007

Lattice data:

O. Kaczmarek, F. Zantow:  
Phys. Rev. D 71 (2005) 054508

- first order deconfinement transition (pure gauge)  
→ cross-over (with quarks)

# PNJL :

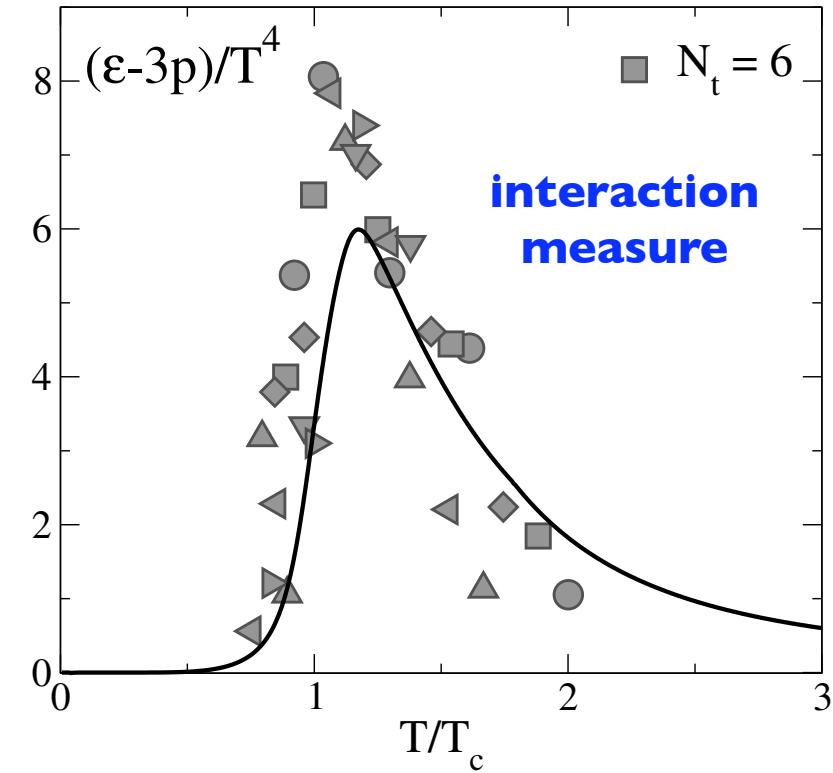
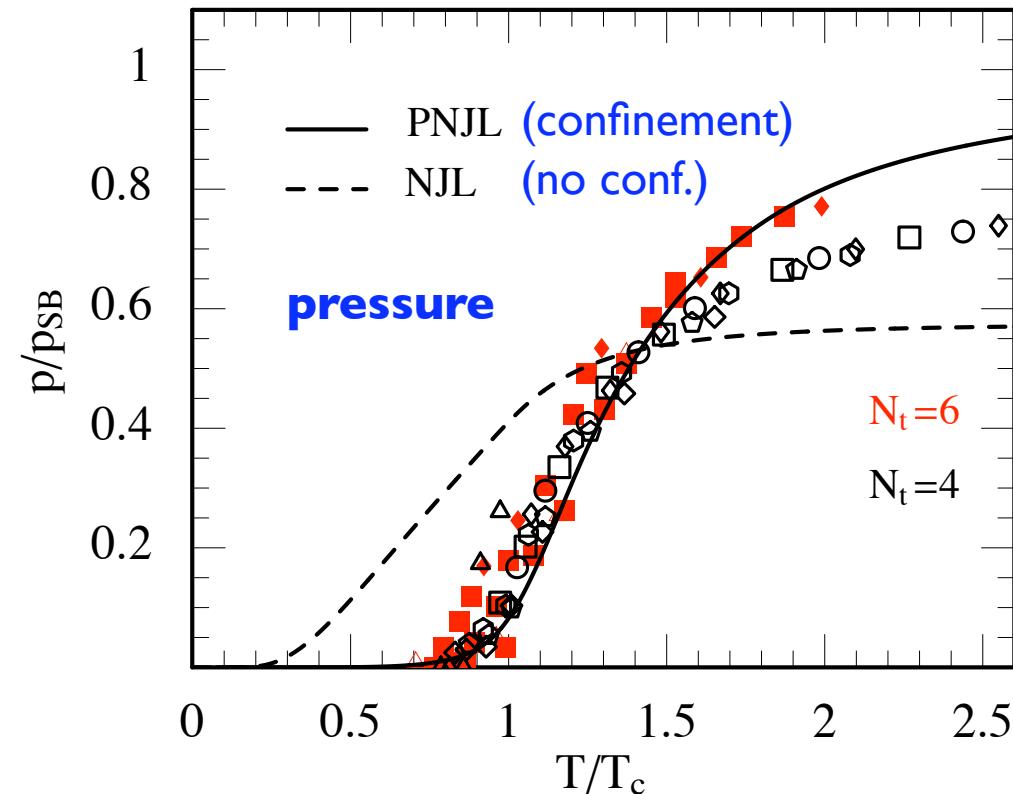
## Comparisons with $N_f = 2$ Lattice Thermodynamics

- PRESSURE and ENERGY DENSITY at zero chemical potential

$$p = -\Omega(T, \mu = 0)$$

$$\varepsilon = T \frac{\partial p(T, \mu = 0)}{\partial T} - p(T, \mu = 0)$$

C. Ratti, M.Thaler, W.W.: Phys. Rev. D 73 (2006) 014019



Lattice data: F. Karsch, F. Laermann, A. Peikert; Nucl. Phys. B 605 (2002) 579

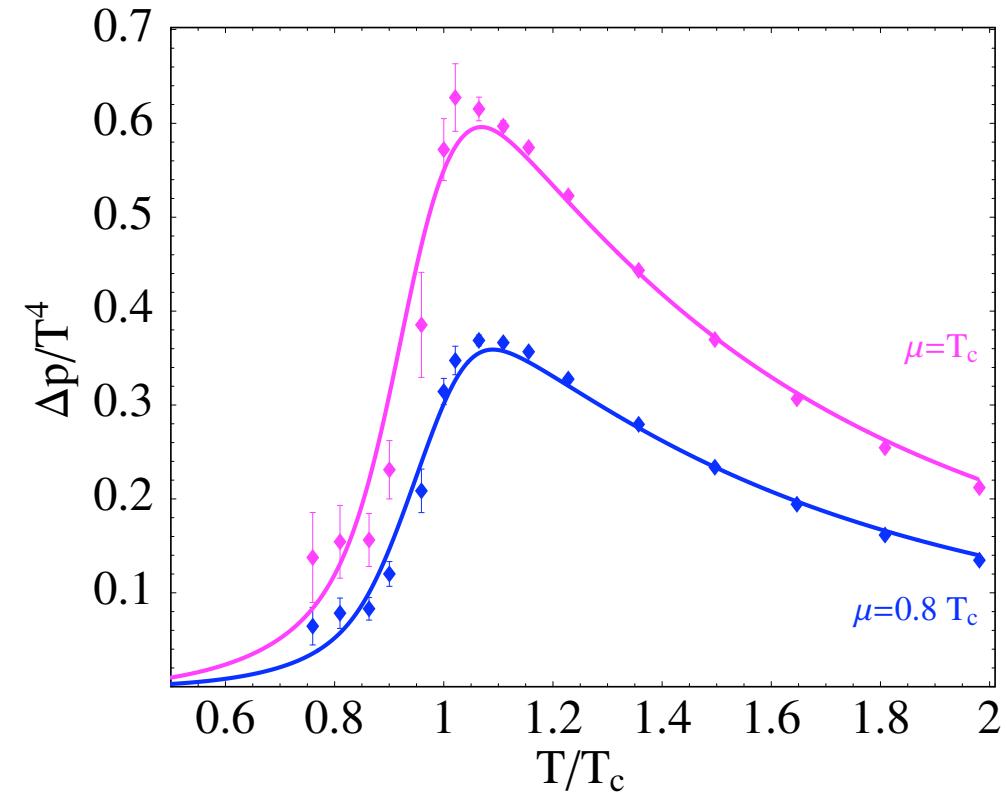
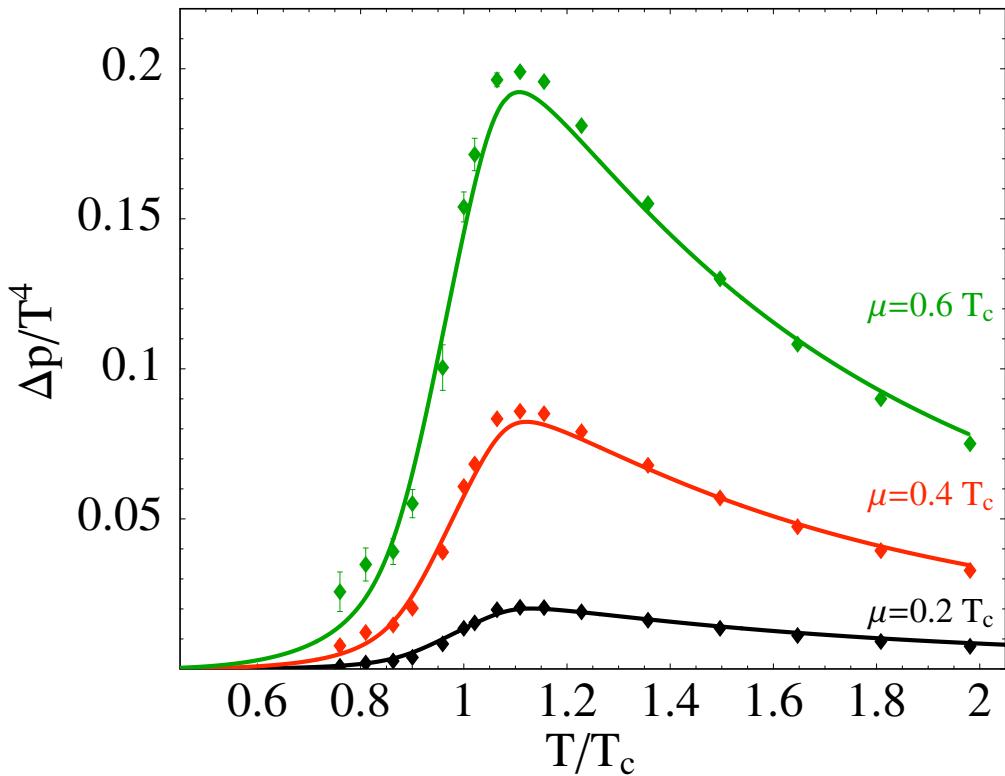
# Results

PART II:

## Non-zero QUARK CHEMICAL POTENTIAL

- Pressure difference:

$$\Delta p(T, \mu) = p(T, \mu) - p(T, \mu = 0)$$



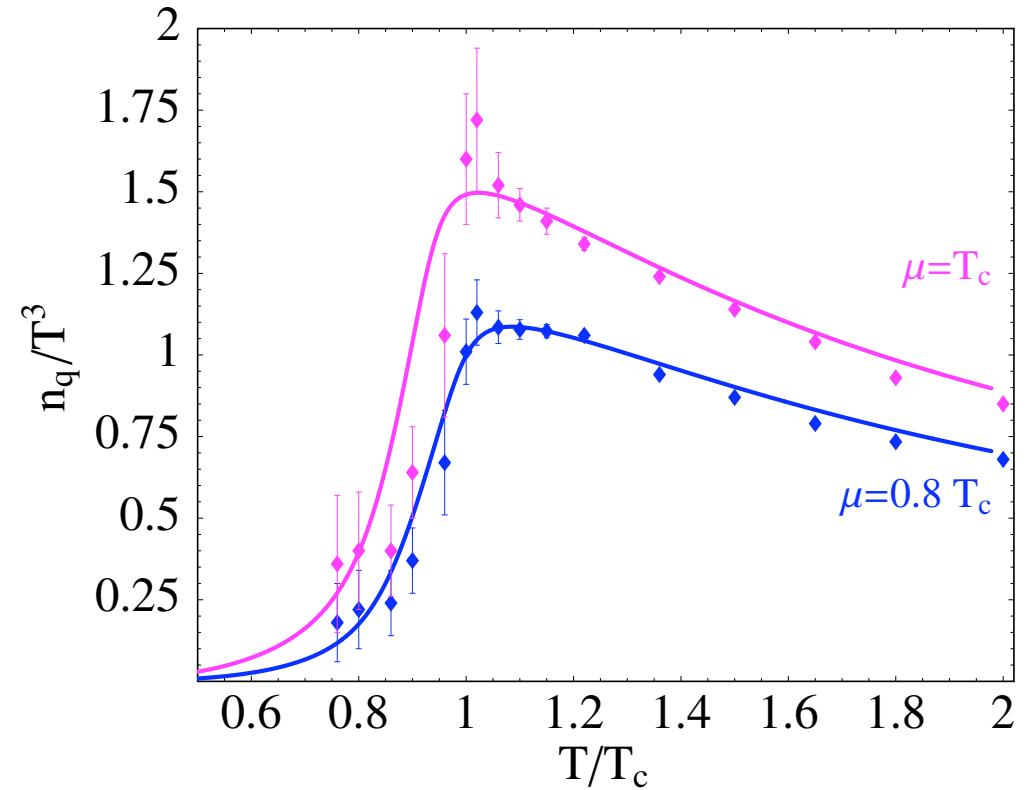
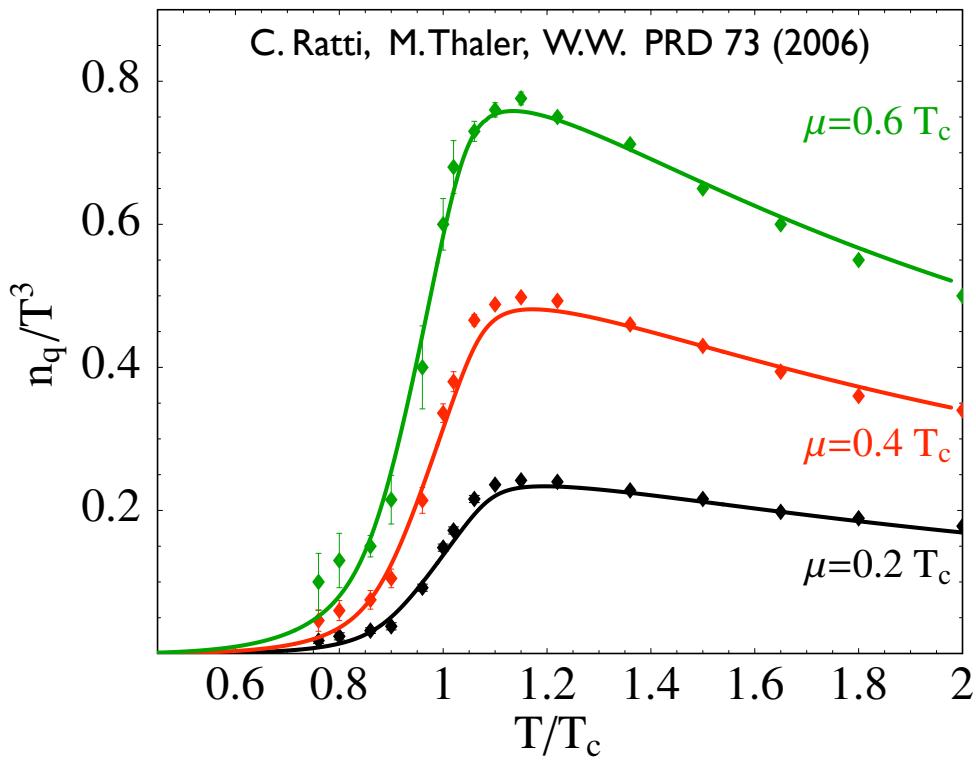
C. Ratti, M.Thaler, W.W.: Phys. Rev. D 73 (2006) 014019

Lattice data: Allton et al. Phys. Rev. D 68 (2003)

# Non-zero QUARK CHEMICAL POTENTIAL (contd.)

- Quark number density:

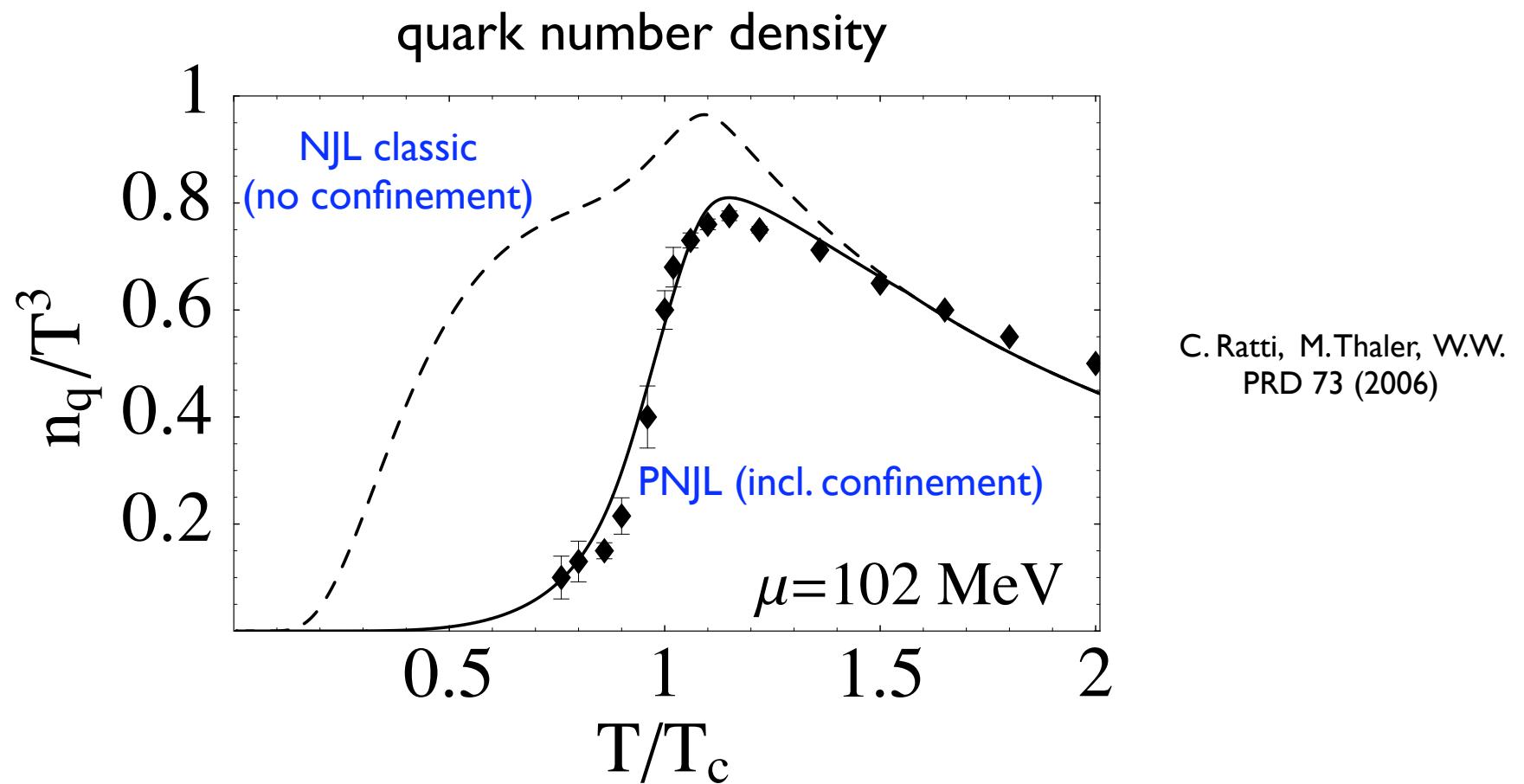
$$n_q(T, \mu) = -\frac{\partial \Omega(T, \mu)}{\partial \mu}$$



Lattice data: Allton et al. Phys. Rev. D 68 (2003)

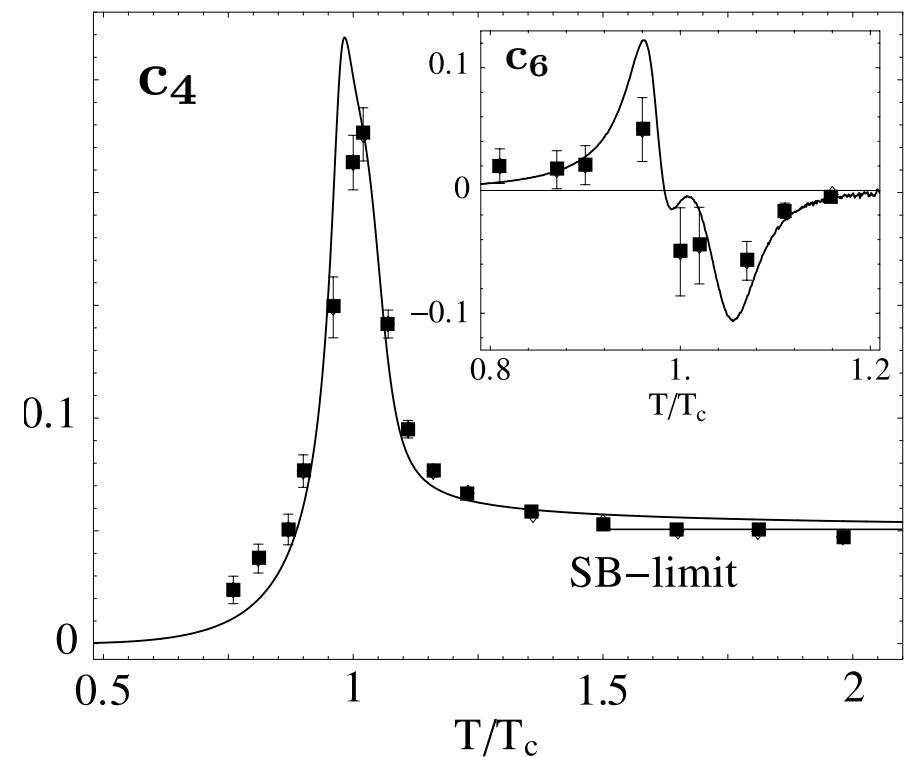
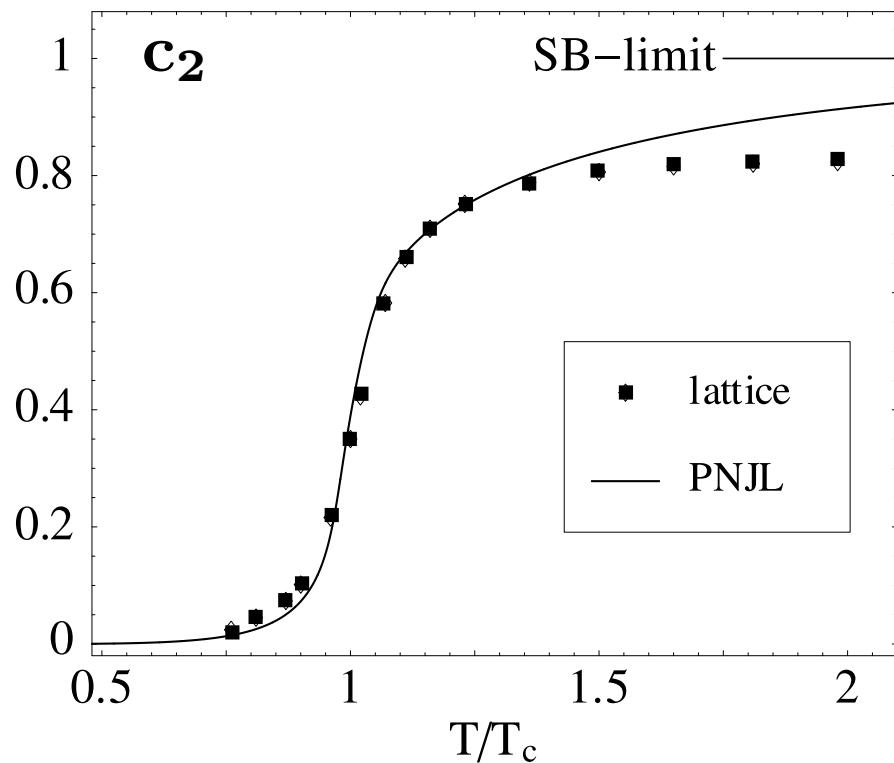
# Non-zero QUARK CHEMICAL POTENTIAL (contd.)

- Role of **CONFINEMENT** (POLYAKOV loop dynamics)



# Non-zero QUARK CHEMICAL POTENTIAL (contd.)

- Taylor expansion of pressure:  $p(T, \mu) = T^4 \sum_n c_n(T) \left(\frac{\mu}{T}\right)^n$



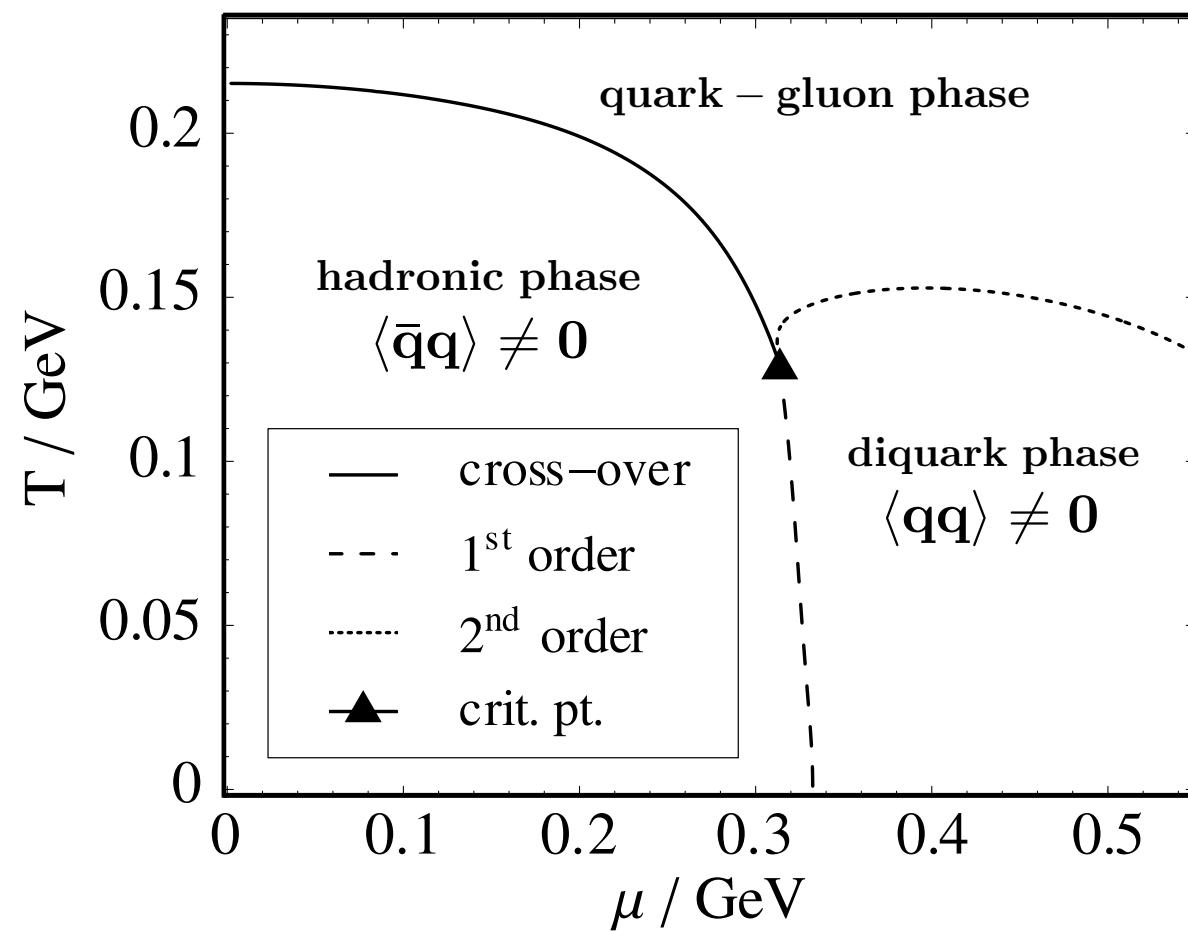
S. Rößner, C. Ratti, W.W.  
Phys. Rev. D 75 (2007) 034007

Lattice:  
C.R.Allton et al.  
Phys. Rev. D 71 (2005) 054508

## 4. PHASE DIAGRAM

- Issues:  $\rightarrow$  Critical point  $\rightarrow$  Diquark (superconducting) phase

S. Rößner, C. Ratti, W.W.: Phys. Rev. D 75 (2007) 034007



... for comparison:

- critical temperature from Lattice QCD

$T_c \simeq 202 \text{ MeV}$   
( 2 flavors )

$T_c = 192 \pm 11 \text{ MeV}$   
( 2+1 flavors )

M. Cheng et al.,  
Phys. Rev. D74 (2006) 054507

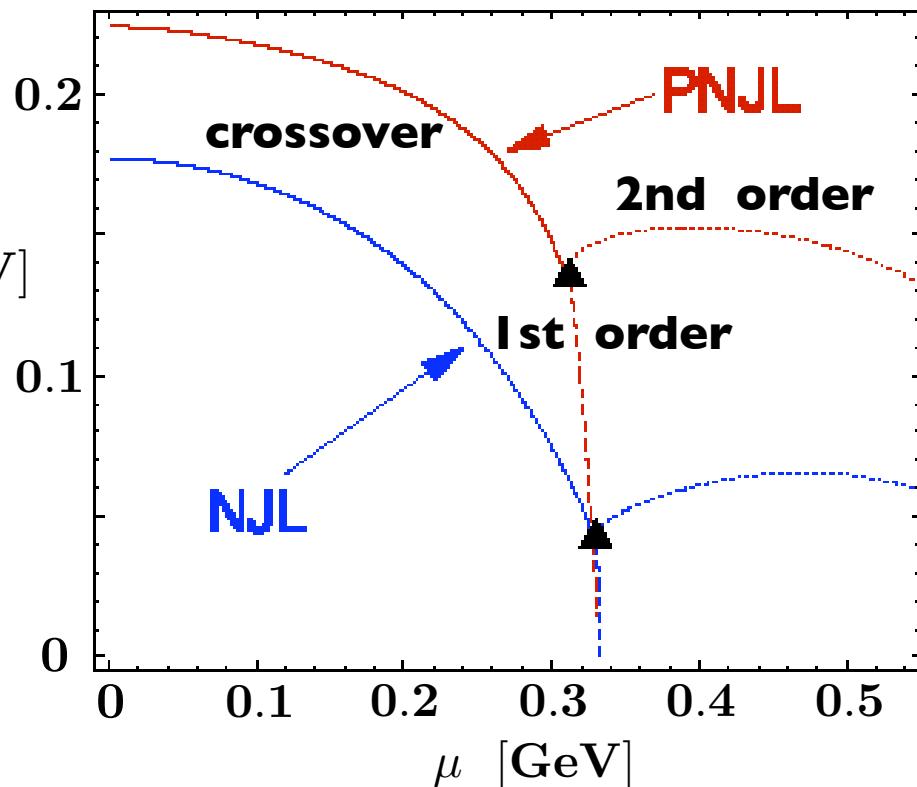
O. Kaczmarek, F. Zantow:  
Phys. Rev. D 71 (2005) 054508

# PHASE DIAGRAM

(contd.)

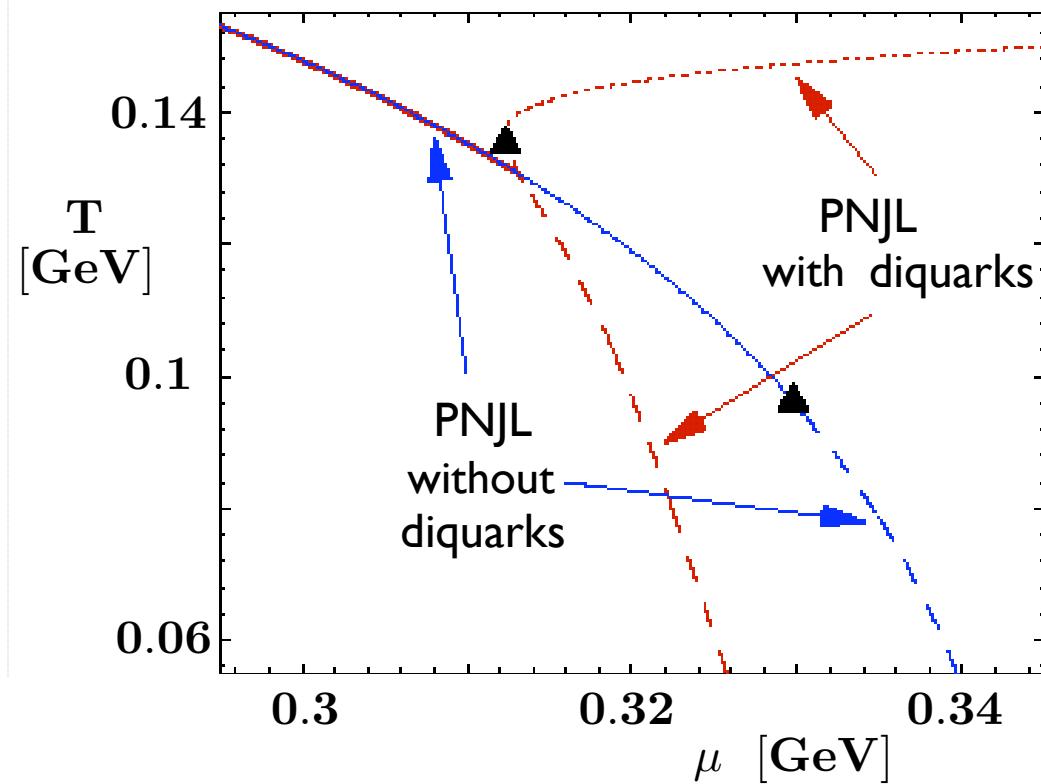
- 

## Effect of Polyakov Loop



- 

## Effect of Diquarks

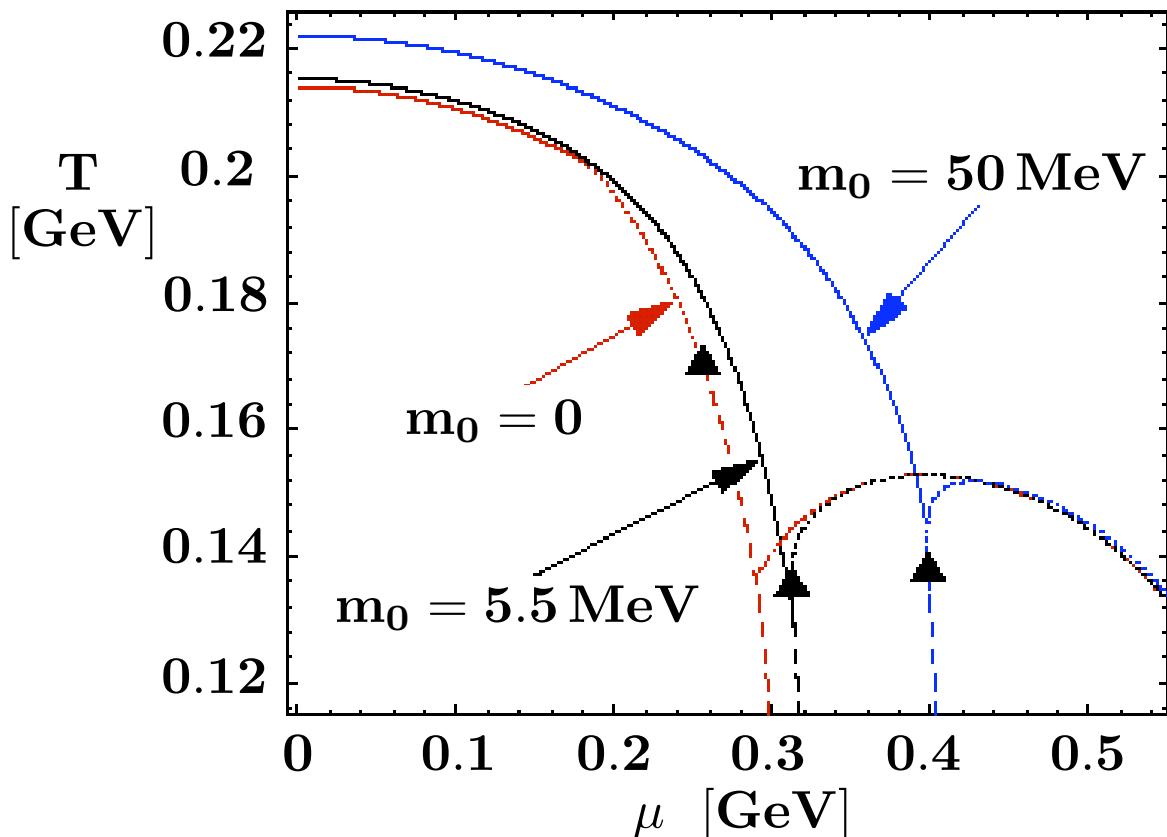


- Location of critical point depends sensitively on active degrees of freedom involved



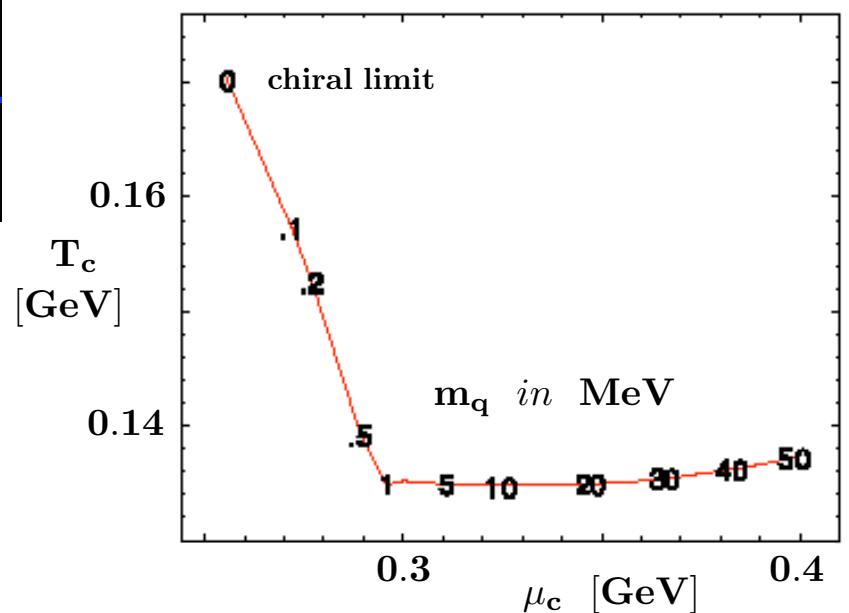
# PHASE DIAGRAM

(contd.)



Location of critical point  
depends sensitively on quark mass

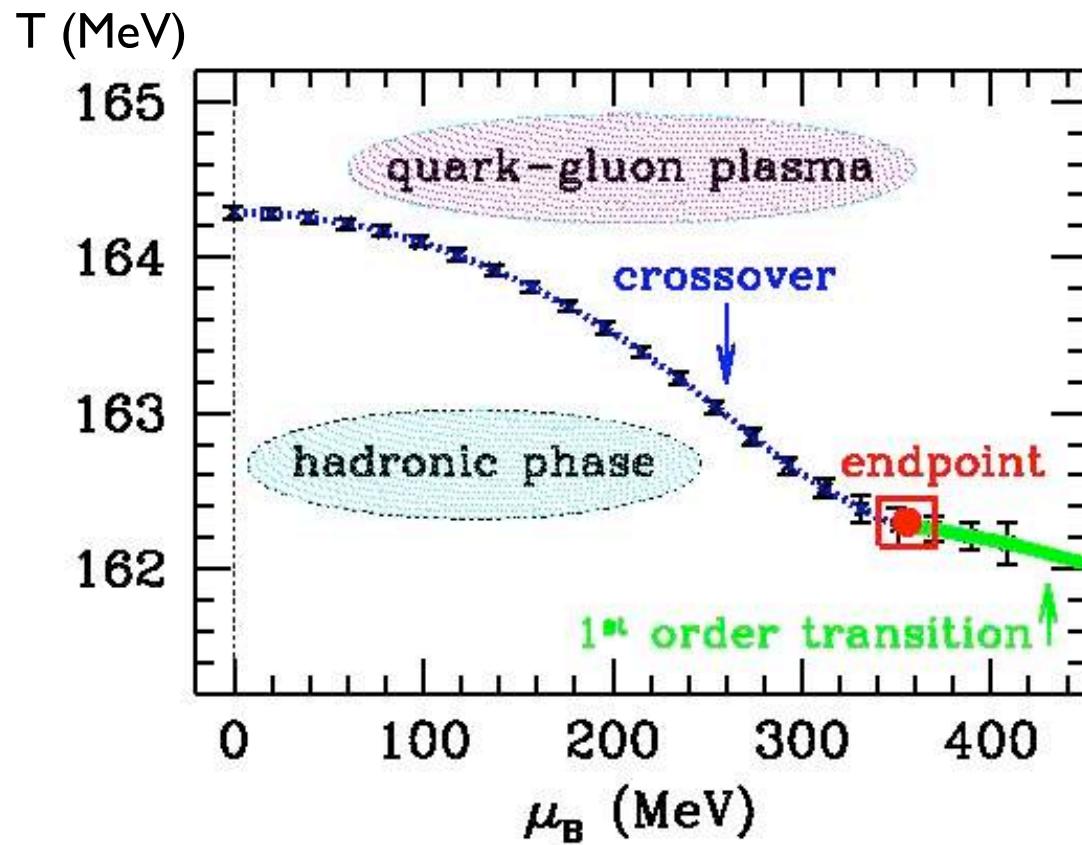
Critical Point  
and  
Quark Mass



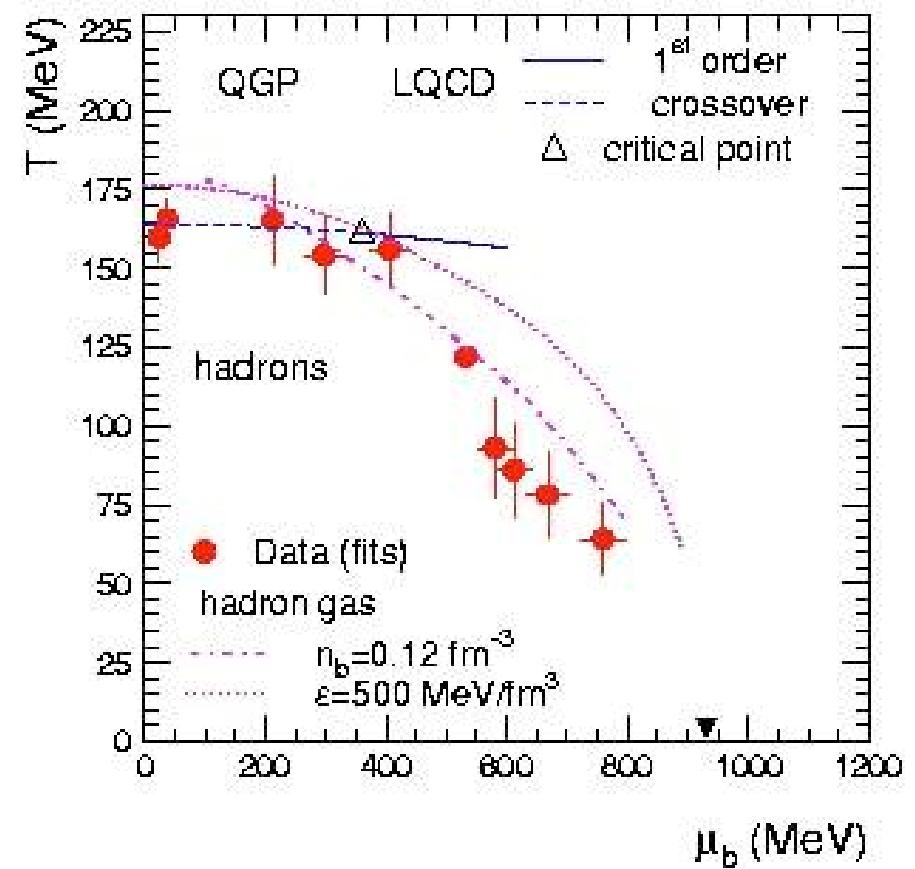
# PHASE DIAGRAM

(contd.)

- Status: Lattice QCD and chemical freezeout phenomenology



Z. Fodor, S. Katz: JHEP 0404 (2004) 050



P. Braun-Munzinger et al. (2006)



## 5. Summary (part I)

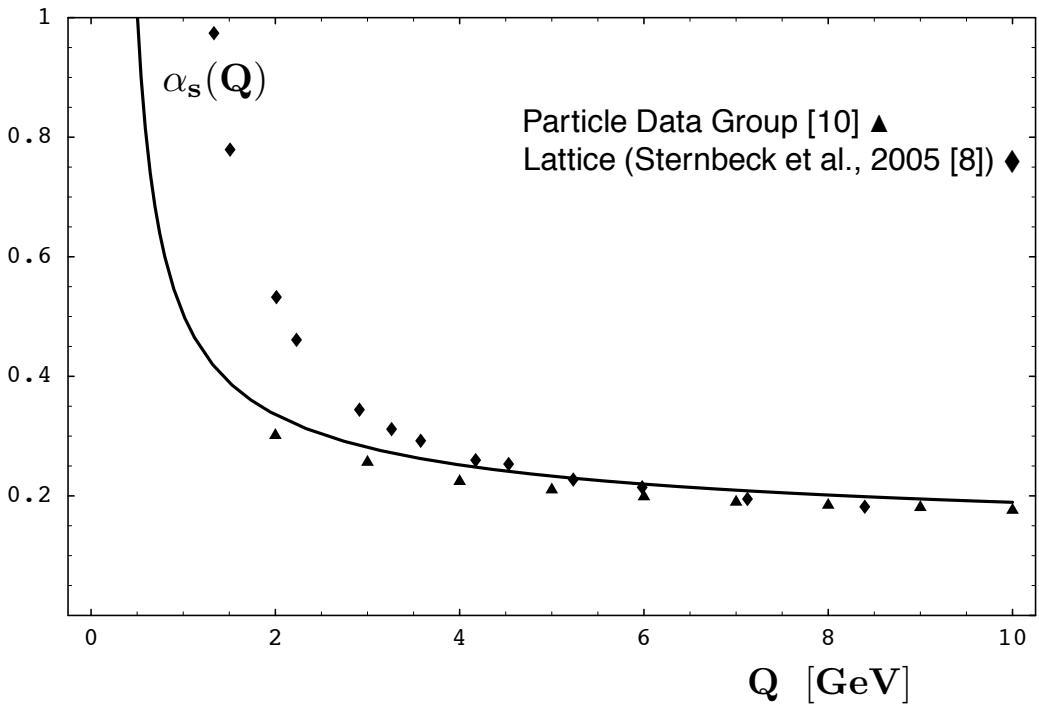
- **QUASIPARTICLE** approach (**PNJL model**) encoding **CHIRAL SYMMETRY** and **CONFINEMENT** in terms of coupled “order parameter” fields  
**(CHIRAL CONDENSATE, POLYAKOV LOOP)**  
surprisingly successful in comparison with  
**QCD THERMODYNAMICS** on the Lattice  
 $(T \lesssim 2 T_c)$

further developments:

- **PNJL** for 2 + 1 flavors (including **DIQUARKS**)
- Establish contacts with high temperature limit  
(Transverse Gluons, "Hard Thermal Loops", Running Coupling)
- Extensions beyond MEAN FIELD ( $\rightarrow$  Simon Rößner)



# 6. Outlook : PNJL with Running Coupling

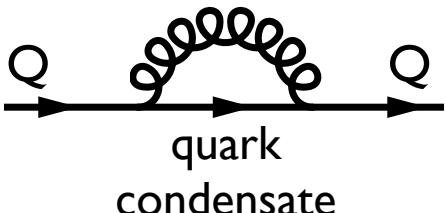


$$M(Q^2) = -G(Q^2)\langle\bar{\psi}\psi\rangle$$

$$\langle\bar{\psi}\psi\rangle = -\frac{N_c N_f}{4\pi^4} \int dQ_4 \int d^3Q \frac{M(Q^2)}{Q^2 + M(Q^2)}$$

at large  $Q$ :

$$G(Q^2) = \frac{2\pi}{3} \frac{\alpha_s(Q^2)}{Q^2}$$

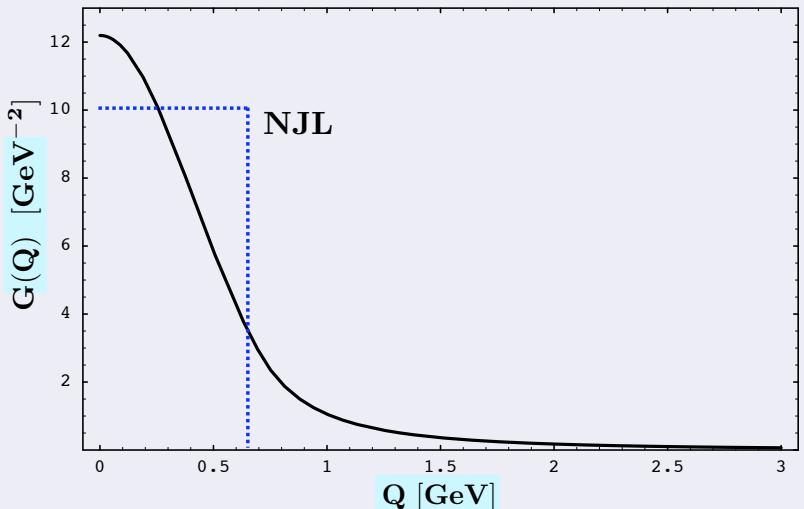


with  
Thomas Hell      Simon Rößner

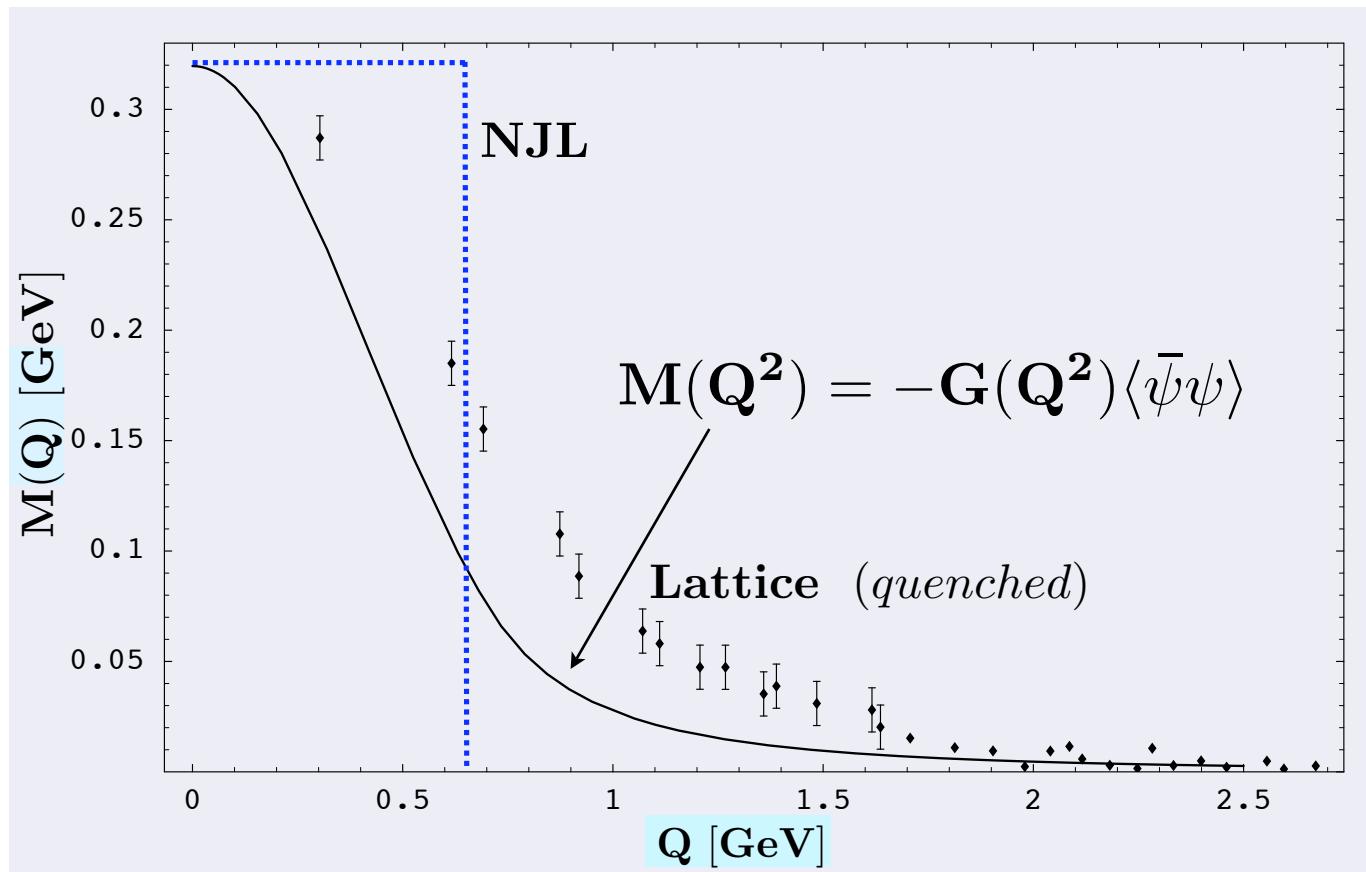
- contacts with PQCD:
- replace NJL cutoff by sliding Q-scale

see also:  
Dyson-Schwinger approach

## Running Nambu-Coupling



# NJL with running coupling: dynamical (constituent) quark mass



- perfect description of **pion properties** and **current algebra**
- in progress:  
implementation of Polyakov loop and  
thermodynamics incl. thermal self-energies

